



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

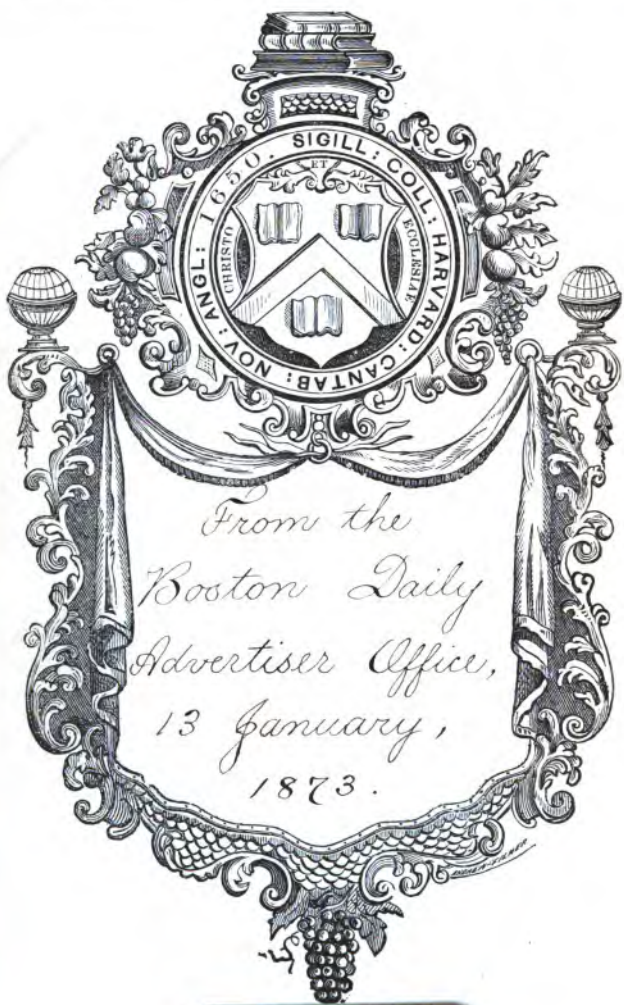
About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

32.35

Edue T 148.72.795

Bd. 1873.





3 2044 097 045 686



Smith, Walter.

3
THE CHILD'S
PRACTICAL GEOMETRY;

BEING

A Series of Elementary Problems

IN DRAWING

PLANE GEOMETRICAL FIGURES,

AS GIVEN IN THE COURSE OF LESSONS IN PUBLIC SCHOOLS.

COMPILED AND ADAPTED BY

WALTER SMITH,

STATE DIRECTOR OF ART EDUCATION, MASSACHUSETTS.

*Special Edition for the Use of Schools, and intended as a Text-Book in Grammar
and High Schools.*

BOSTON:
JAMES R. OSGOOD AND COMPANY,
(LATE TICKNOR & FIELDS, AND FIELDS, OSGOOD, & CO.)

1872.

ANNOUNCEMENT.

MESSRS. JAMES R. OSGOOD & CO. have the pleasure of announcing that they have arranged with Mr. WALTER SMITH, Professor of Art Education in the Boston Normal School of Art, and State Director of Art Education, Massachusetts, for the publication of his series of text-books on Drawing.

This series will be the most complete and comprehensive one ever published in America, and will be adapted to schools of all grades.

The Drawing-Books will be prepared on an entirely new and original plan, and will embrace a thorough explanation and illustration of the principles of

Free-Hand Drawing,

Model and Object Drawing,

Perspective, Geometrical, and Mechanical Drawing.

Copies of the work on Free-Hand Drawing will be published shortly.

JAMES R. OSGOOD & CO., Publishers,

124 TREMONT STREET, BOSTON.

THE CHILD'S
PRACTICAL GEOMETRY;

BEING

A Series of Elementary Problems

IN DRAWING

PLANE GEOMETRICAL FIGURES,

AS GIVEN IN THE COURSE OF LESSONS IN PUBLIC SCHOOLS.

COMPILED AND ADAPTED BY

WALTER SMITH,

STATE DIRECTOR OF ART EDUCATION, MASSACHUSETTS.

*Special Edition for the Use of Schools, and intended as a Text-Book in Grammar
and High Schools.*

BOSTON :

JAMES R. OSGOOD AND COMPANY,

(LATE TICKNOR & FIELDS, AND FIELDS, OSGOOD, & CO.)

1872.

Entered according to Act of Congress, in the year 1872, by JAMES R. OSGOOD & Co., in the Office
of the Librarian of Congress at Washington.

Edge T 148, 72, 795
1873, Jan. 13.

~~Math 5258.72~~ From the

Daily Advertiser Office.

PREFACE.

ALTHOUGH existing books on Geometrical Drawing are numerous, and some of them excellent, yet in a wide experience I have found no book quite adapted to the requirements of students in schools of art, still less suitable as a class-book for use in schools for general education.

My object in compiling this work has been to supply a deficiency felt, I believe, generally by art-masters who have an extensive field for instruction. Several years of experience in teaching classes, both in schools of art and in other schools, as well as in superintending the teaching of drawing in primary and adult schools, have enabled me to try experiments in systematizing the arrangement of geometrical problems, and to obtain conclusions based on the results of such experiments. The experience thus obtained has been embodied in this work.

In the "*School of Art Practical Geometry*," I have endeavored to produce an elementary text-book, which should contain problems for the construction of such geometrical figures as are of most frequent use to the designer or artisan: the knowledge of at least all the problems contained in it will be a necessary preliminary to the study of Perspective, Mechanical Drawing, and Architectural Drawing. Repetition of methods for arriving at the same result has been as much as possible avoided, and the simplest method, if practically accurate, has been adopted, even where other methods more mathematically accurate could have been used, when counterbalanced with the evil of puzzling and complicated working.

The importance of a knowledge of Geometrical Drawing at the commencement of a study of drawing is paramount; and, though many persons learn to draw well without a knowledge of it, yet I have never known a case where a student did not progress more satisfactorily in his studies after a course of practical geometry. I have also been led to believe that such an elementary course of geometrical drawing as I have compiled would be of use in schools as a preparation for the study of Euclid. The care required in drawing the figures with accuracy, and the explanations given at the blackboard demonstrations, will be found to lead to a correct knowledge of geometric forms; and it seems only rational that upon the successful practice of constructing such forms might be built a habit of thinking and reasoning accurately about their nature.

The present part comprises only the elementary problems of a work to be shortly issued, containing a wider range of study, and greater variety of problems.

DIRECTIONS TO THE PUPIL.

BEFORE commencing the *Problems*, it will be necessary that the *Definitions* be committed to memory, and the figures illustrating them drawn with tolerable accuracy.

A problem in geometrical drawing is naturally divided into three parts, which, avoiding mathematical expressions, may be called, 1st, the conditions; 2d, the working; 3d, the result. Thus, for example, in Prob. No. 10, which is "to draw a line parallel to a given line AB at the distance CD from it," the first part, or conditions, being the lines AB and CD; the second part, or working, being the arcs described; the third part, or result, being the line EF.

To distinguish these separate features is very necessary for a clear understanding of a problem; and, to facilitate this, I have used three distinct kinds of lines in the illustrations:—

The first, or conditions, being the *thin lines*, _____

The second, or working lines, being dotted lines, _____

The third, or result lines, being *thick lines*, _____

So that, with a single glance at the illustration, the pupil may see, 1st, what had to be done; 2d, how it has been done; 3d, the figure when done.

When working from the blackboard, the pupil need make no difference between the conditional lines and the working lines; but the result lines should be made darker than the others. But in working the problems in his lesson-book afterwards (which should always be done as soon as possible after the lesson), the lines of the problem should either be distinguished by the three lines as used in the illustrations, — i.e., thin, thick, and dotted lines; or, what is still better, the conditions should be in *blue* color, the working in *red*, the result in *black*. For red and blue, use crimson lake and Prussian blue colors, not red and blue ink, which change color after use.

It will be seen, by referring to the illustrations, that I have used both letters and numerals in the description: this is also to help to make the problems more intelligible. The letters are used for points in the conditions and results, the numerals in the working. When points in the working will form part of the description of the result, then letters are used; but when it is necessary to use points in the working, which form no part of the result, then numerals are used. The numerals may also assist in showing how the work is done, by indicating the steps taken: thus, point 1 will be the first point in the working, either assumed or obtained; point 2 will be a consequence of the use of point 1; the point 3 will result from the use of 2; and so on.

In arranging the problems, I find it better to put them in pairs which either treat of the same figure, or have a similar result. Thus, by referring to the sheet of problems commencing 31, it will be seen that 31 is to make an equilateral triangle on a given base; 32, in a given circle; 33, to make a square on a given base; 34, in a given circle; and the same principle of arrangement may be observed throughout all the first part of the book, and is as much as possible adhered to in the latter part also.

The Index of the Problems and Definitions is to be used as a test of the student's power of remembering what he has learnt. After working out all the problems from the book, he should again work them out from the index, without referring to the text or illustrations. The questions on the definitions should be answered by sketches, and by writing them from memory.

DEFINITIONS.

LINES AND ANGLES.

Def. 1. A *point A*, is a term used to express a position.

EXAMPLE. — The lines 1 2 and 3 4 cross or intersect each other in point A.

LINES.

A Line is usually described by two letters, as AB; or two figures, such as 1 2, as in the last example.

Def. 2. A *right* or *straight* line AB, is the shortest line that can be drawn between two points, A and B.

Def. 3. A *curved* line AB, is one which is otherwise than straight.

Def. 4. *Parallel* lines AB and CD, are those which are everywhere equally distant from each other; and which, therefore, if continued, can never meet.

ANGLES.

Def. 5. An *angle* BAC is formed when two straight lines, BA and CA, meet at the same point A.

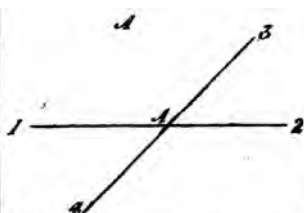
Def. 6. There are three kinds of angles. 1. Right angles; 2. Obtuse angles; 3. Acute angles.

Def. 7. A *right angle* is formed thus: when one straight line AB, standing on another straight line CD, makes the two angles ABC and ABD equal to each other, each angle is said to be a right angle. When two lines form right angles, each line is said to be perpendicular to the other.

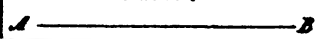
Def. 8. An *obtuse* angle ABC is that which is greater than a right angle, DBC.

Def. 9. An *acute* angle ABC is that which is less than a right angle, DBC.

DEF. 1.



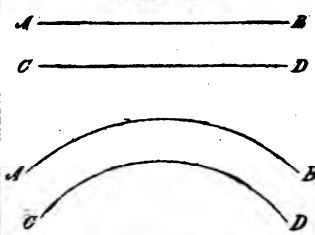
DEF. 2.



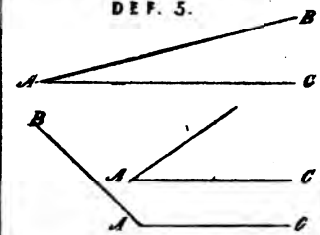
DEF. 3.



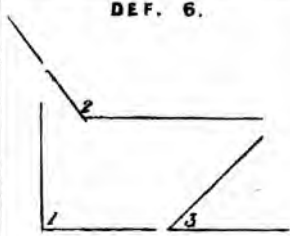
DEF. 4.



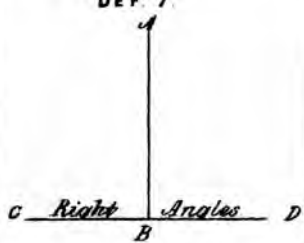
DEF. 5.



DEF. 6.



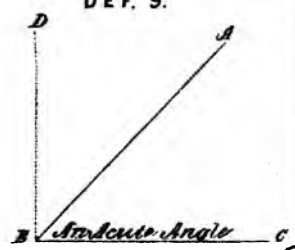
DEF. 7.



DEF. 8.

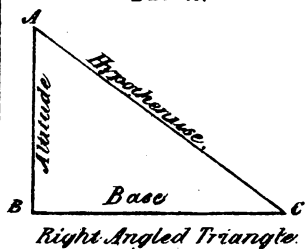


DEF. 9.

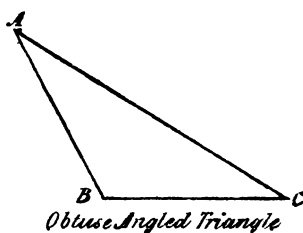


4.

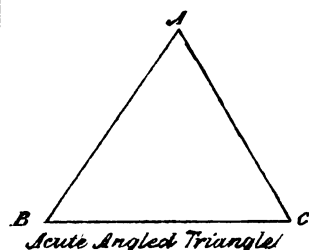
DEF. 11.



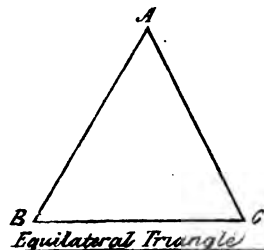
DEF. 12.



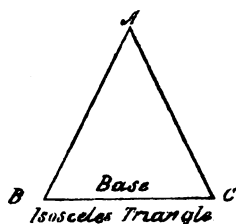
DEF. 13.



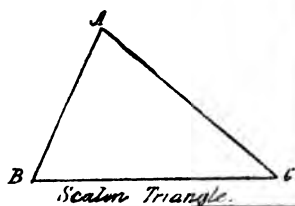
DEF. 14.



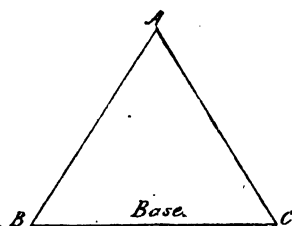
DEF. 15.



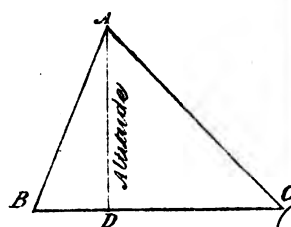
DEF. 16.



DEF. 17.



DEF. 18.



TRIANGLES.

Def. 10. A figure having three sides and three angles is called a *triangle*. There are six different kinds of triangles, three of which are named from the character of their angles, and three from the proportion of their sides.

The triangles named from the character of their angles are, —

Def. 11. A *right-angled triangle* ABC is that which has one of its angles a right angle B.

Def. 12. An *obtuse-angled triangle* ABC has one of its angles an obtuse angle B.

Def. 13. An *acute-angled triangle* ABC has all its three angles acute angles A and B and C.

The triangles named from the proportion of their sides are, —

Def. 14. An *equilateral triangle* ABC is that which has its three sides equal.

Def. 15. An *isosceles triangle* ABC is that which has two of its sides equal. The third side BC is called its base.

Def. 16. A *scalene triangle* is that which has its three sides unequal.

Def. 17. The three lines forming a triangle are usually described as the base and two sides. The base is the line on which the triangle is supposed to rest. Thus, in the triangle ABC, BC is the base, and AC and AB the two sides. The angle opposite the base (at point A) is called the *vertical angle*, and the other two angles are called the *angles at the base* (at points B and C).

Def. 18. The *altitude* or *height* of a triangle AD is the distance from the vertical angle A to the base BC, or the base produced, and is formed by drawing a straight line from the vertical angle, until it cuts the base at right angles, or the base produced.

QUADRILATERALS OR FOUR-SIDED FIGURES.

A figure of four sides and angles is called a quadrilateral, or quadrangle.

There are six quadrilaterals, four of which are also called parallelograms :

A *parallelogram* is a quadrilateral whose opposite sides are parallel and equal.

The names of the four parallelograms are, —

Def. 19. A square ABCD has four equal sides, and all its angles right angles.

Def. 20. An *oblong* or *rectangle* ABCD has its opposite sides equal and parallel, and all its angles right angles.

Def. 21. A *rhombus* ABCD has four equal sides; but its angles are not right angles.

Def. 22. A *rhomboid* ABCD has its opposite sides equal and parallel, but its angles are not right angles.

The quadrilaterals which are not parallelograms are, —

Def. 23. A *trapezium* ABCD is a figure having four sides, no two of which are parallel.

Def. 24. A *trapezoid* ABCD is a figure having four sides, two only of which are parallel.

POLYGONS.

Figures having more than four sides are called polygons.

Polygons are, —

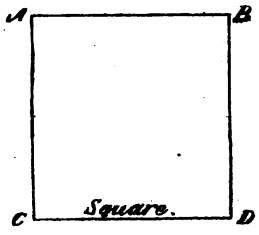
Def. 25. *Regular* polygons if all their sides and all their angles are equal ABCDEF.

Def. 26. *Irregular* polygons when their sides and angles are unequal ABCDEF.

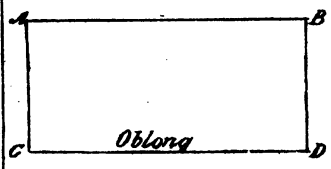
Def. 27. A polygon having five sides is called a Pentagon.

"	"	six	"	"	Hexagon.
"	"	seven	"	"	Heptagon.
"	"	eight	"	"	Octagon.
"	"	nine	"	"	Nonagon.
"	"	ten	"	"	Decagon.
"	"	eleven	"	"	Undecagon.
"	"	twelve	"	"	Duodecagon.

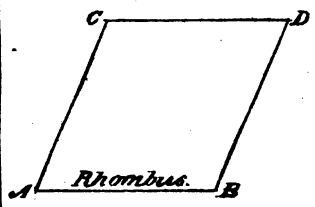
DEF. 19.



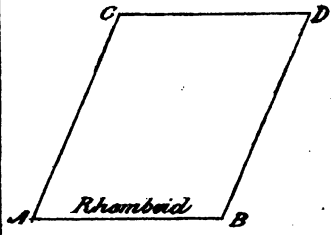
DEF. 20.



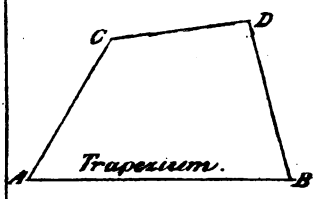
DEF. 21.



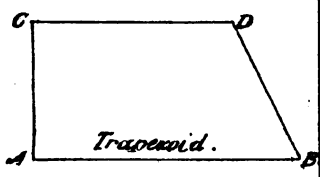
DEF. 22.



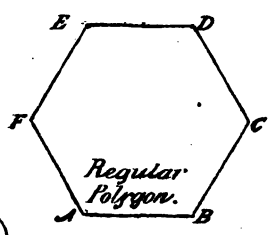
DEF. 23.



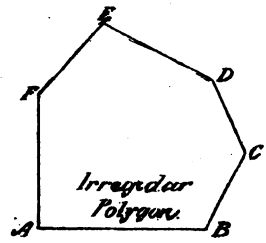
DEF. 24.



DEF. 25.

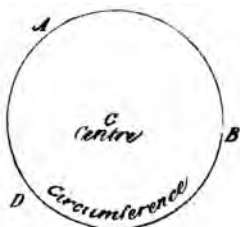
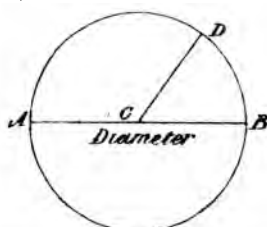
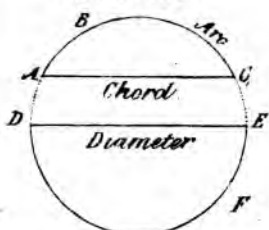
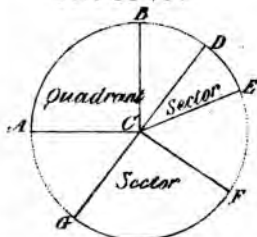


DEF. 26.

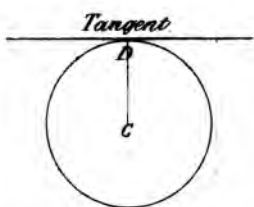
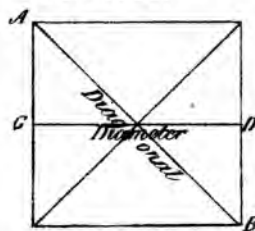
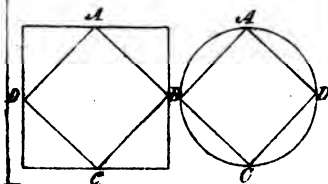
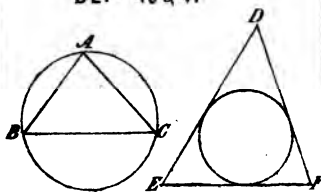


8.

DEF. 28.

DEF^s 29 & 30DEF^s 31 & 32.DEF^s 33 & 34

DEF. 35.

DEF^s 36 & 37.DEF^s 38 & 39.DEF^s 40 & 41

THE CIRCLE.

- Def. 28.* A *circle* ABD is a figure bounded by one curved line called the circumference, every part of which is equally distant from a point C within it, called the centre.
- Def. 29.* A *diameter* AB is a straight line drawn through the centre of the circle, and terminated at both ends by the circumference.
- Def. 30.* A *radius* CD is a straight line drawn from the centre to the circumference.
- Def. 31.* An *arc* ABC is a portion of a circumference, and a *chord* is a straight line AC joining the extremities of an arc.
- Def. 32.* A *semicircle* DEF is half a circle, bounded by half the circumference and a diameter.
- Def. 33.* A *quadrant* ABC is a quarter of a circle, bounded by the two radii AC and BC, and a quarter of the circumference. In a quadrant the two radii form a right angle.
- Def. 34.* A *sector* is a portion of a circle contained by two radii and a part of the circumference. Thus CDE and FCG are both sectors.
- Def. 35.* A *tangent* AB is a straight line which touches the circumference of a circle in a point, without cutting it. A tangent is always at right angles to a radius DC drawn from the point of contact D.
- Def. 36.* A *diagonal*, in a quadrilateral or four-sided figure, is a line joining opposite angles, as AB.
- Def. 37.* A *diameter* of a parallelogram is a line drawn through the centre, parallel to two of the sides, as CD.
- Def. 38.* A figure is said to be inscribed within another when all the angles of the inscribed figure are on the sides of the figure in which it is inscribed, as ABCD.
- Def. 39.* A figure is said to be inscribed in a circle when all the angles of the figure are in the circumference of the circle, as ABCD.
- Def. 40.* A circle is said to be described about a figure when the circumference of the circle passes through all the angles of the figure about which it is described, as ABC.
- Def. 41.* A figure is said to be described about a circle when each side of the figure touches the circumference of the circle, as DEF.

PROBLEMS.

PROB. I. — To bisect a given line AB, either straight or part of a circle.

With A as centre, and any radius greater than half AB, describe an arc; with B as centre, and the same radius, intersect the arc in 1 and 2; from 1 draw a right line to 2, and it will bisect AB in C.

PROB. II. — At point C in a given line AB, to draw a line perpendicular to it.

With C as centre, and any radius, cut the line AB in points 1 and 2; with 1 as centre, and a longer radius, describe an arc; and with 2 as centre, with the same radius as at 1, cut the arc in point 3; draw 3C, which is the required perpendicular.

PROB. III. — To draw a line perpendicular to a given line AB from any point C outside and opposite to it.

[Use the description to the last problem for this one also, except that the radii 1 3 and 2 3 may be shorter.]

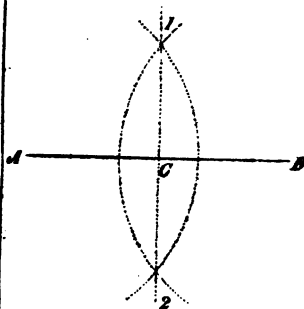
PROB. IV. — To draw a line perpendicular to a given line AB from a point C nearly or quite over the extremity of AB.

From the point C draw a straight line to any point 1 in AB; bisect the line C1 in point 2; with 2 as centre, and 2 1 as radius, describe an arc cutting AB in point 3; draw the line C3, which will be the required perpendicular.

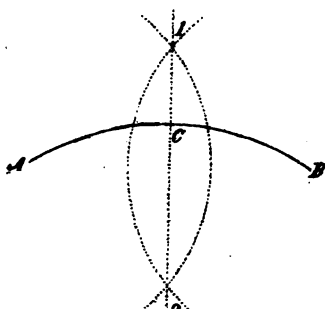
PROB. V. — To erect a perpendicular at the extremity of a given line AB.

With B as centre, and any radius B1, describe an arc from AB cutting in 1; and with 1 as centre, and the same radius, intersect it in 2; with 2 as centre, same radius, describe an arc over B; draw a line from 1 through 2 to intersect the last arc in 3; draw a line 3B, which is the required perpendicular.

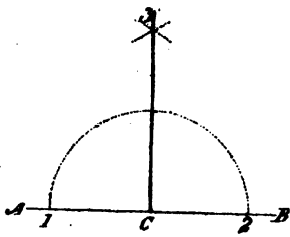
PROB. 1.



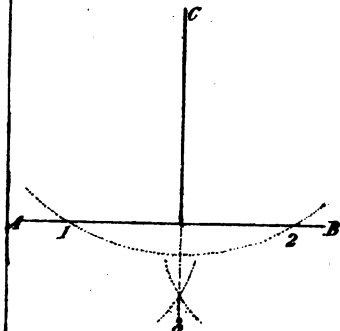
PROB. 1.



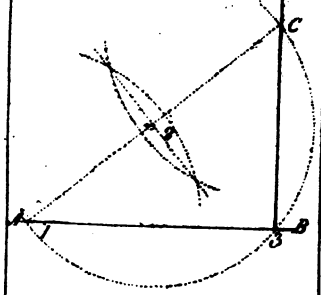
PROB. 2.



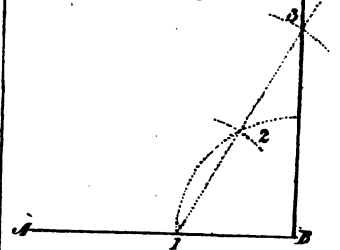
PROB. 3.



PROB. 4.

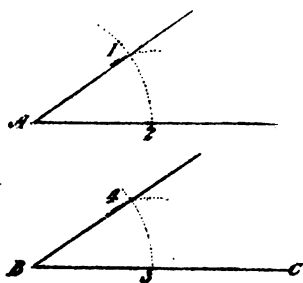


PROB. 5.

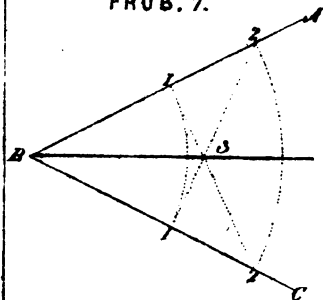


12.

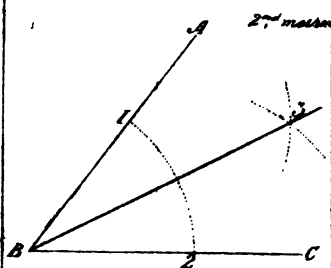
PROB. 6



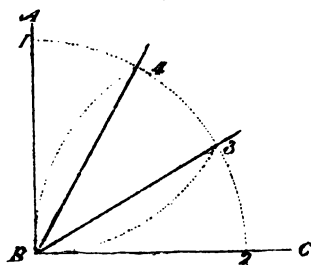
PROB. 7.



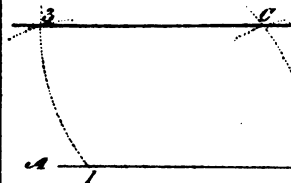
PROB 7



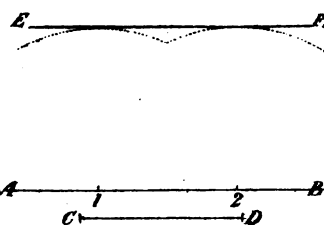
PROB. 8.



PROB 9



PROB. 10.



PROB. VI. — To make an angle equal to a given angle A at a given point B in a line BC.

With A as centre, and any radius, cut the sides of angle A in points 1 and 2. From centre B, same radius, describe an arc from BC, cutting it in 3. Make 3 4 equal to 2 1; draw B4: 4BC will be the required angle.

PROB. VII. — To bisect a given angle ABC.

With B as centre, any radius, describe an arc cutting AB and CB in points 1 1; same centre and greater radius, cut the lines again in 2 2; draw lines 1 2 and 1 2, cutting in 3; draw a line from B through 3; B3 bisects the angle.

Second Method. — With B as centre, and any radius, cut the sides of the angle in points 1 and 2; with 1 and 2 as centres, same radius, describe arcs intersecting in 3; draw line B3 bisecting the angle.

PROB. VIII. — To trisect a right angle ABC.

With B as centre, any radius, cut the sides of the angle in two points by the arc 1 2; with 1 and 2 as centres, same radius, intersect the arc in points 3 and 4; draw lines B3 and B4 trisecting the right angle.

PROB. IX. — To draw a line through a point C parallel to a given line AB.

Take any point 1 in AB as centre, and with 1C as radius describe an arc to cut AB in 2; with centre C, same radius, describe a similar arc from 1, and make 1 3 equal to 2C; draw a line through 3C, which will be the line required.

PROB. X. — To draw a line parallel to a given line AB at a distance from it equal to CD.

Take any two points 1 and 2 on AB, and with 1 and 2 as centres, and radius CD, describe arcs above the line; draw a line EF tangential to or touching the arcs: EF will be the required parallel.

PROB. XI. — *To construct an oblong having two given sides AB and CD.*

Make EF equal to AB, and erect a perpendicular FG at F, by Prob. 5, equal to CD; with centre E, radius CD, describe an arc above E; and with centre G, radius AB, intersect the arc in H; draw GH and EH: EFGH is the required oblong.

PROB. XII. — *To construct an oblong, its diagonal AB and one side CD being given.*

Bisect AB in 1; with 1 as centre, 1A radius, describe a circle; with A as centre, CD radius, cut the circle in E; and with B as centre, same radius, cut the circle in F; draw AEBF, the required oblong.

PROB. XIII. — *To construct a rhombus, having a given diagonal AB, and a given side CD.*

With centres A and B, radius CD, describe arcs intersecting in E and F; draw AEBF, the required rhombus.

PROB. XIV. — *To construct a rhombus, having a given side AB and a given angle C.*

At the point A make an angle equal to C, by Prob. 6; make AD equal to AB; and with centres D and B, radius AB, describe arcs cutting in E; draw ADEB, the required rhombus.

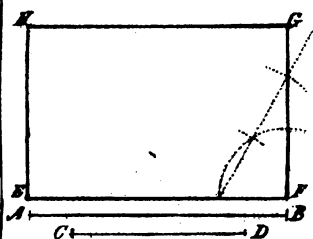
PROB. XV. — *To construct a triangle of three given lines A, B, and C.*

Make DE equal to A; with centre D, radius B, describe an arc, and with centre E, radius C, cut the arc in F; draw DEF, the required triangle.

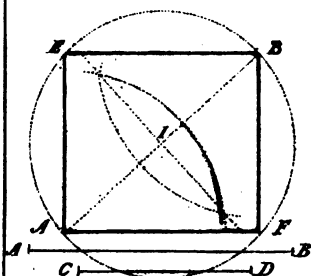
PROB. XVI. — *To construct a triangle, its base AB and the angles at the base A and B being given.*

Make line CD equal to AB; make angle C equal angle A, and angle D equal angle B, by Prob. 6; continue the sides until they meet in point E.

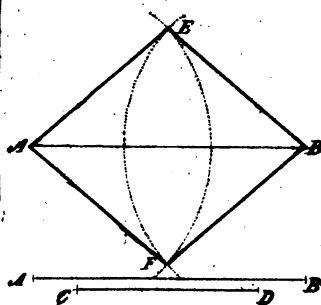
PROB. 11.



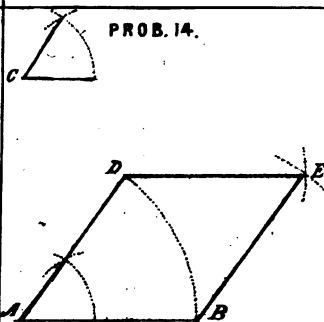
PROB. 12.



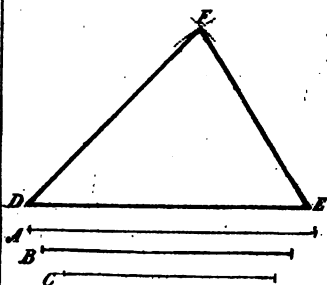
PROB. 13.



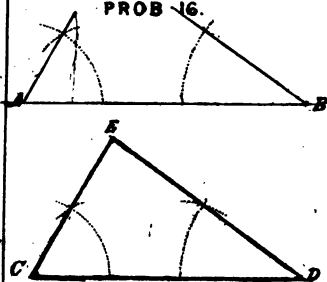
PROB. 14.



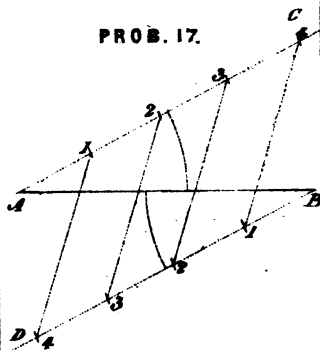
PROB. 15.



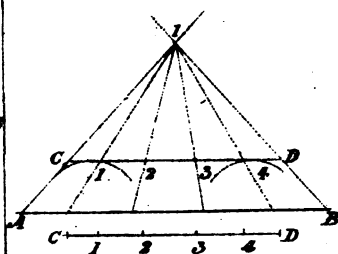
PROB. 16.



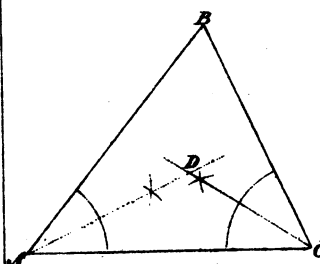
PROB. 17.



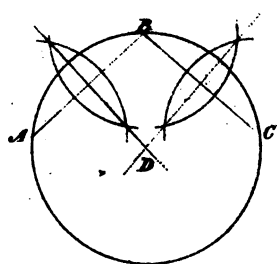
PROB. 18.



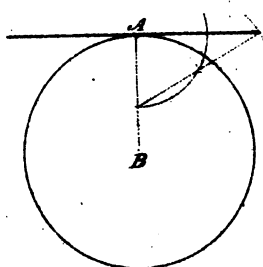
PROB. 19.



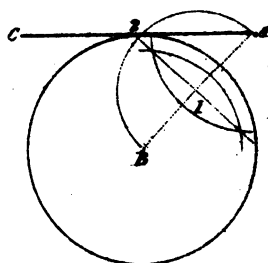
PROB. 20.



PROB. 21.



PROB. 22.



PROB. XVII. — *To divide a given line AB into any number of equal parts.*

Suppose the line is required to be divided into five. Draw a line AC at any angle with AB, and draw the line BD at the same angle with AB, by Prob. 6; commencing at A, mark off on AC the number of points, less one, that AB is to be divided into; that is, set off four equal parts of any length, as 1 2 3 4. From B, mark off on BD the same number of similar parts; join 1 4, 2 3, 3 2, &c., by lines dividing AB into five equal parts.

PROB. XVIII. — *To divide a line AB into the same proportions as a given divided line CD.*

Place CD above AB, parallel to AB, by Prob. 10; draw lines AC, BD, until they meet in point 1; draw lines from 1 through the divisions on CD to AB, which will then be divided proportionately to CD.

PROB. XIX. — *To find the centre of a given triangle ABC.*

Bisect two of the angles by Prob. 7; continue the two bisecting lines until they meet in point D, which is the centre required.

PROB. XX. — *To find the centre of a circle, ABC.*

Draw two chords, AB, BC; bisect them by Prob. 1; draw the bisecting lines until they cut in point D, which is the centre of the circle.

PROB. XXI. — *To draw a tangent to a circle at a given point of contact A.*

Find the centre of the circle by Prob. 20; draw a radius AB; at point A erect a perpendicular to AB by Prob. 5: it is the tangent required.

PROB. XXII. — *To draw a tangent to a circle from a given point A outside the circumference.*

Find the centre of the circle by Prob. 20; draw a line from A to the centre of the circle B; bisect AB in point 1; with centre 1, radius 1A, describe a semicircle on AB, cutting the given circle in point 2; draw a line AC through point 2, which will be the required tangent.

PROB. XXIII. — *Within a given triangle ABC, to inscribe a circle.*

Bisect angles A and B by Prob. 7, to find the centre of the triangle, point 1. From 1 draw a line perpendicular to any side of the triangle, as 1 2, by Prob. 3; with 1 as centre, and 1 2 radius, describe the required circle.

PROB. XXIV. — *About or outside a given triangle, to describe a circle.*

Bisect two sides of the triangle by Prob. 1, continuing the bisecting lines until they meet in point 1; with 1 as centre, radius 1A or 1B, or 1C, describe the required circle.

PROB. XXV. — *To make a trapezium equal to a given trapezium ABCD.*

Draw a line EF equal to AB; at F construct an angle equal to the angle at B by Prob. 6, making FG equal to BC; with G as centre, CD as radius, describe an arc; and with E as centre, AD as radius, cut it in point H: EFHG will be equal to ABCD.

PROB. XXVI. — *To make an irregular polygon equal to a given irregular polygon ABCDE.*

Draw a line FG equal to AB; with centre F, radius AD, describe an arc; and with centre G, radius BD, cut it in 1. With centre I, radius DE, describe an arc, and with centre G, radius BE, cut it in K; with centre K, radius EC, describe an arc, and with centre G, radius BC, cut it in H: FGHKI will be equal to ABCDE.

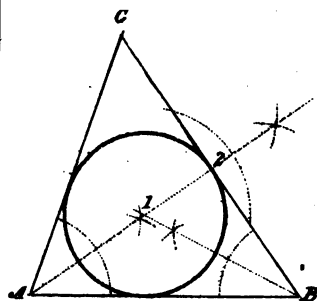
PROB. XXVII. — *To draw a line from a point C, lying outside a line AB, to make an angle with the line AB equal to a given angle D.*

Through C draw a line parallel to AB, by Prob. 9; at the point C on the line make an angle equal to D, by Prob. 6; produce the line forming the angle to cut AB in E: CEB will be equal to D.

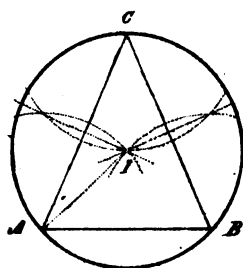
PROB. XXVIII. — *To draw a tangent to a given arc AB at a given point of contact C, when the centre of the arc cannot be used.*

With C as centre, any radius, cut the arc in two places, points 1 and 2; draw line 1 2. Through the point C draw a line DE parallel with 1 2: DE is the required tangent.

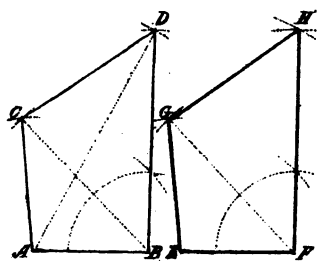
PROB. 23.



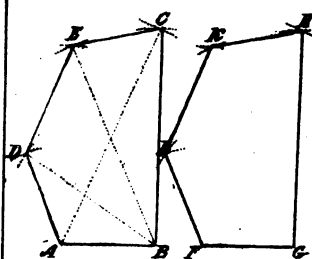
PROB. 24.



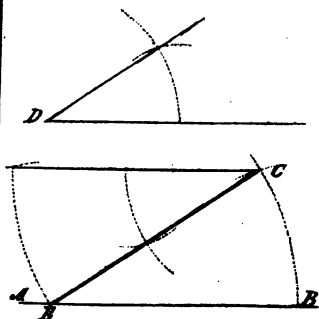
PROB. 25.



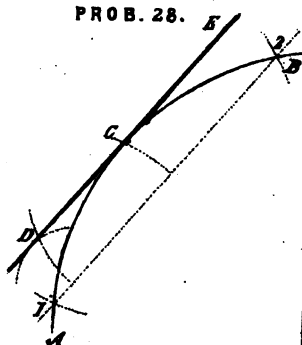
PROB. 26.



PROB. 27.

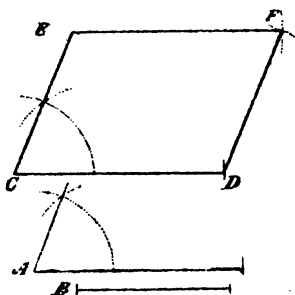


PROB. 28.

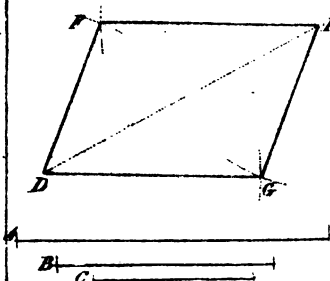


20.

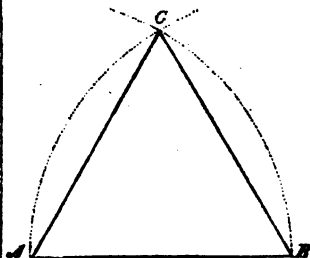
PROB. 29.



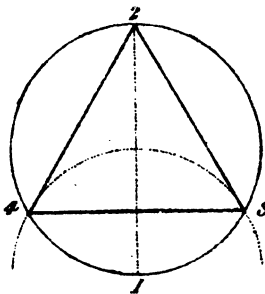
PROB. 30.



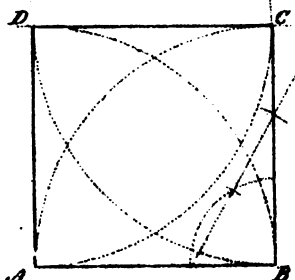
PROB. 31.



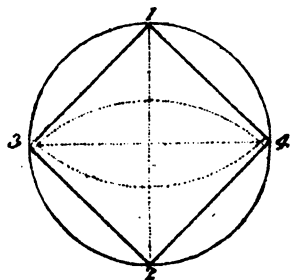
PROB. 32.



PROB. 33.



PROB. 34.



PROB. XXIX. — To construct a rhomboid, having its two sides A and B and an angle given.

Draw a line CD equal to A, and at C construct an angle equal to the angle at A by Prob. 6, making the line CE equal to B; with centre E, radius A, describe an arc, and with centre D, radius B, cut it in F: join EF, FD, for the required rhomboid.

PROB. XXX. — To construct a rhomboid, its two sides B and C, and its diagonal A, being given.

Make DE equal to A; with centres D and E, radius B, describe an arc on each side of DE; with radius C, from D and E as centres, cut the first arcs in F and G: draw DFEG, the required rhomboid.

PROB. XXXI. — On a given base AB, to construct an equilateral triangle.

With A as centre, radius AB, describe an arc, and with centre B, same radius, intersect it in C; draw AC and BC: ABC will be the required triangle.

PROB. XXXII. — Within a given circle, to inscribe an equilateral triangle.

Draw a diameter 1 2. With centre 1, and the radius of the given circle as radius, describe an arc to cut the circumference in points 3 and 4; draw 2 3, 3 4, 4 2: 2 3 4 is the required triangle.

PROB. XXXIII. — On a given base AB, to construct a square.

At point B erect a perpendicular by Prob. 5, making BC equal to AB; from centre A, radius AB, describe an arc, and from centre C, same radius, cut it in D; join CD, DA.

PROB. XXXIV. — Within a given circle, to inscribe a square.

Draw a diameter 1 2; bisect it by a perpendicular, as in Prob. 1; continue the bisecting line both ways to cut the circle in points 3 and 4: draw 1 3, 3 2, 2 4, 4 1, the required square.

PROB. XXXV. — On a given base AB, to construct a regular hexagon.

With centre A, radius AB, describe an arc, and with centre B, same radius, cut it in C; with centre C, same radius, describe a circle, and from B mark AB, round its circumference, in points DEFG: draw BD, DE, EF, FG, GA.

PROB. XXXVI. — Within a given circle, to inscribe a regular hexagon.

Draw a diameter AB. With centres A and B, and the radius of the circle as radius, describe an arc, cutting the circumference in CD and EF: join AD, DF, FB, BE, EC, and CA.

PROB. XXXVII. — On a given base AB, to construct any regular polygon.

Suppose the polygon to be a pentagon: continue the base BA to the left; with centre A, radius AB, describe a semicircle; divide it into as many parts as the polygon required has sides, — in this case five; draw a line from A to the second division of the semicircle A2; bisect A2 and AB by Prob. 1, continuing the bisecting lines to meet in point C; with C as centre, radius CA, describe a circle, and mark off the length AB from B, round the circumference, and join the adjacent points: ABDE2 is the polygon.

[NOTE. — This method applies to every polygon, the line A2 being always the second side of the polygon. The student should practise this problem on polygons of seven, eight, and nine sides.]

PROB. XXXVIII. — Within a given circle, to inscribe any regular polygon.

Find the centre of the circle by Prob. 20; draw a diameter 1 2; divide it by Prob. 17 into as many equal parts as there are sides to the polygon, — suppose five, for example; with centre 1, radius 1 2, describe an arc, and centre 2, same radius, cut it in point 3; draw a line from 3 through the second division from 1, and continue it to cut the circumference of the circle in point A; draw line 1 A: it will be one side of the required polygon; set off 1 A round the circle in points ABCD, and join the points AB, BC, CD, D1.

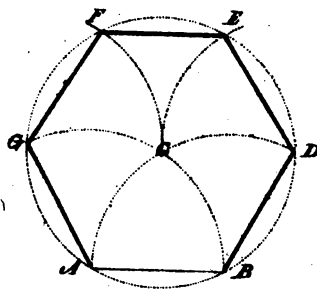
PROB. XXXIX. — On a given altitude AB, to construct an equilateral triangle.

By Prob. 5, erect a perpendicular at B to line AB. With A as centre, any radius, cut AB in point 1; and with centre 1, same radius, describe an arc to cut it again in point 2. With centre 2, same radius, cut the last arc in points 3 and 4. Draw lines from A through 3 and 4 to cut the perpendicular in C and D: ACD is the required triangle.

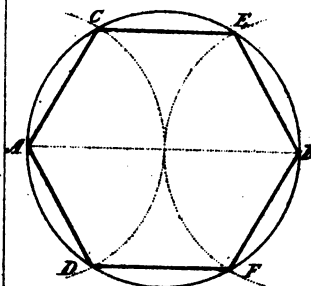
PROB. XL. — To construct an isosceles triangle, having a given altitude AB and a given base CD.

At point B erect a perpendicular by Prob. 5; bisect CD and mark off half of it on the perpendicular on each side of B, in points E and F; join AE and AF: AEF is the required triangle.

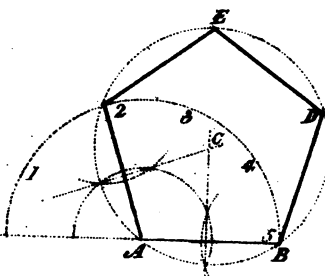
PROB. 35.



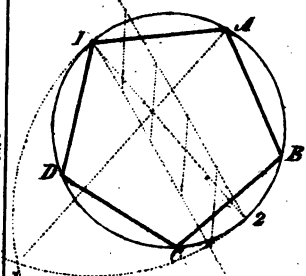
PROB. 36.



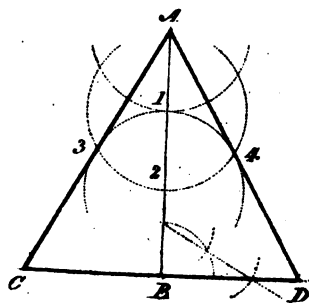
PROB. 37.



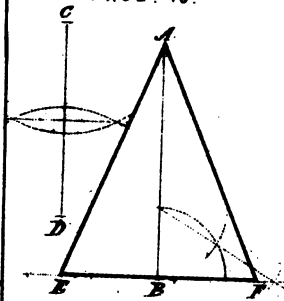
PROB. 38.



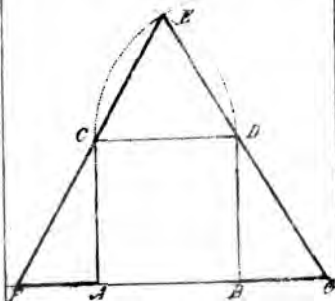
PROB. 39.



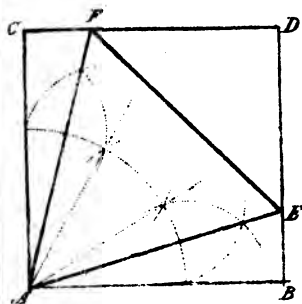
PROB. 40.



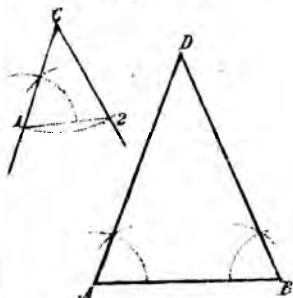
PROB. 41.



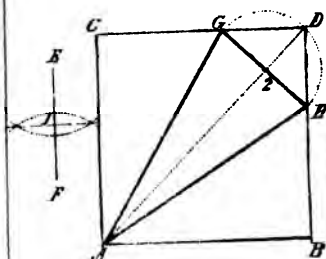
PROB. 42.



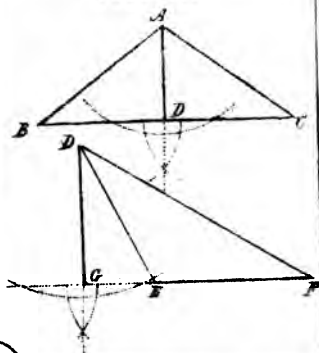
PROB. 43.



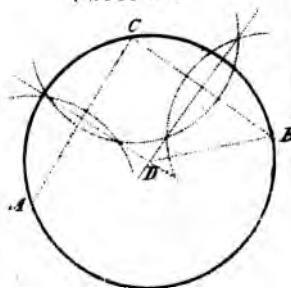
PROB. 44.



PROB. 45.



PROB. 46.



PROB. XLI. — *About a given square ABCD, to describe an equilateral triangle.*

Produce the base AB outwards on both sides; with centre C, radius CD, describe an arc, and with centre D, same radius, cut it in E; draw EC, ED, and produce them to cut the produced base in FG: EFG is the required triangle.

PROB. XLII. — *Within a given square ABCD, to inscribe an equilateral triangle.*

Trisect the angle at A by Prob. 8; bisect the two outer angles of the three into which the right angle is trisected; draw the bisecting lines of the outer angles to cut the sides of the square in F and E; draw EF: AEF is the required triangle.

PROB. XLIII. — *To construct an isosceles triangle on a given base AB, and having a given vertical angle C.*

With the angular point C as centre, and any radius, cut the side of the angle in 1 and 2; draw line 1 2; at points A and B, by Prob. 6, make angles equal to the angles at 1 and 2; produce the lines forming the angles to cut in D: ABD will be the triangle required.

PROB. XLIV. — *Within a given square ABCD, to inscribe an isosceles triangle, having a given base EF.*

Draw a diagonal AD; bisect EF in point 1; from D mark off the distance E1 on the diagonal AD in point 2; with 2 as centre, 2D radius, cut the sides of the square BD and CD in points G and H; draw GH, HA, GA: GHA will be the inscribed triangle.

PROB. XLV. — *To find the altitude of any triangle ABC.*

When the vertical angle A is over the base. — By Prob. 3, let fall a perpendicular to the base BC, cutting it in D: AD is the altitude.

When the vertical angle is not over the base, as in triangle DEF. — Produce the base EF in the direction of the vertical angle, and from D let fall the perpendicular to the produced base, cutting it in G: DG will then be the altitude.

PROB. XLVI. — *To describe a circle through three given points ABC, which are not to be in the same straight line.*

Draw the line AC, CB; bisect them, producing the bisecting lines to cut in D: D will be the centre of the circle. With centre D, radius DB, describe the required circle.

PROB. XLVII. — *Within a given triangle ABC, to inscribe a square.*

Draw the altitude of the triangle AD by Prob. 45; at C erect a perpendicular C1 by Prob. 5, making it equal to the base BC; draw line 1D, cutting the side AC in E; from E let fall a perpendicular to the base BC, cutting it in F; EF is one side of the square; from F mark off the length FE on the base BC in G, and from G with the same length cut AB in H; draw EH and GH: EFGH is the square.

PROB. XLVIII. — *Within a given regular hexagon, to inscribe a square.*

Draw a diagonal AD; bisect the diagonal by a perpendicular in point 1 by Prob. 1, to cut the hexagon in points 2 and 3; bisect two adjacent angles 2IA and 2ID, producing the lines to cut the hexagon in GHK; draw GK, KH, HI, and IG: GHIK is the square.

PROB. XLIX. — *On a given diagonal AB to construct a square.*

Bisect AB in 1, and with centre 1, radius 1A, describe a circle to cut the bisecting line in C and D; draw CA, CB, AD, BD. ACBD is the required square.

PROB. L. — *Within a given square ABCD, to inscribe an octagon.*

Draw the diagonals AC, DB, cutting in I; with centres ABCD, radius AI, describe arcs to cut the square in EF, GH, IK, LM; draw MG, FK, HL, IE: GFKHЛИEM is the required octagon.

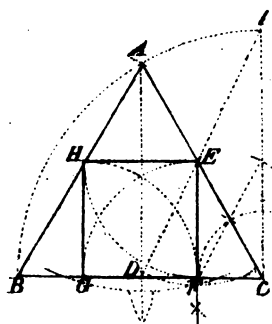
PROB. LI. — *To inscribe a square in a given rhombus ABCD.*

Draw the two diagonals AB, CD, cutting in point 1; bisect two adjacent angles DIA, DIB, producing the bisecting lines to cut the rhombus in EFGH; draw EF, EH, HG, FG: EFGH is the square.

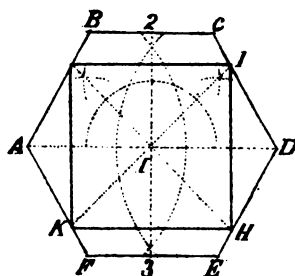
PROB. LII. — *To inscribe a circle in a given rhombus ABCD.*

Draw the two diagonals, cutting in 1; from 1 draw a line perpendicular to any side, DC, cutting it in point 2; with centre 1, radius 1 2, describe the required circle.

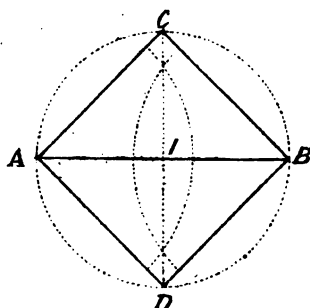
PROB 47.



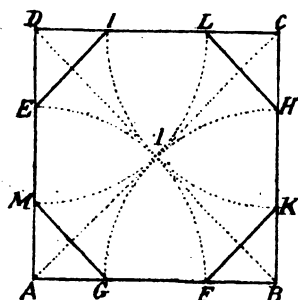
PROB 48.



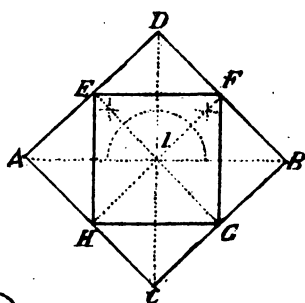
PROB 49.



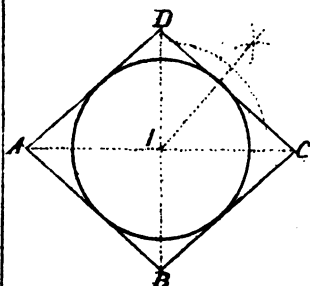
PROB 50.



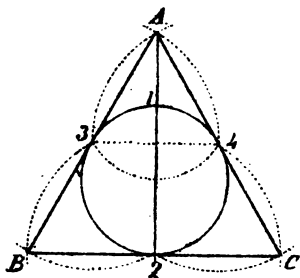
PROB 51.



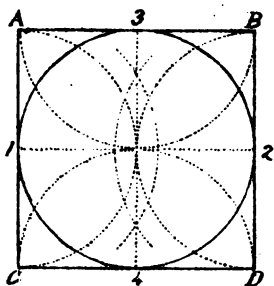
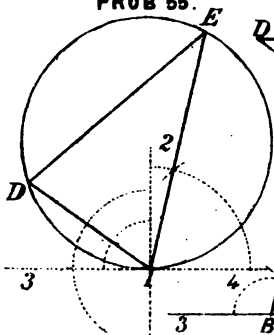
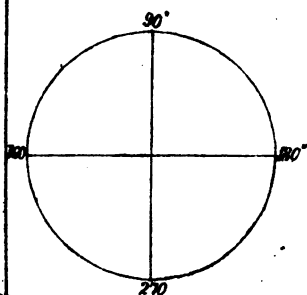
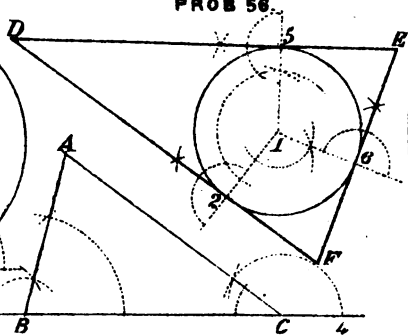
PROB 52.



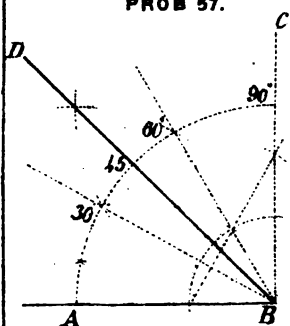
PROB 53.



PROB 54.

**PROB 55****PROB 56**

PROB 57.



PROB. LIII. — *About a given circle, to describe an equilateral triangle.*

Draw a diameter 1 2; with centre 1, and the radius of the given circle as radius, cut it in 3 4; from the centres 2, 3, 4, radius 3 4, describe arcs cutting in ABC; draw AB, AC, BC, the required triangle.

PROB. LIV. — *About a given circle, to describe a square.*

Draw two diameters 1 2, 3 4, perpendicular to each other; with centres 1, 2, 3, 4, and the radius of the circle as radius, describe arcs intersecting in ABCD; draw AB, BD, DC, CA: ABCD is the required square.

PROB. LV. — *Within a given circle to inscribe a triangle similar to a given triangle ABC.*

Find the centre of the circle, and at any point 1 draw a tangent to it 3 4; at point 1 on the tangent make the angle 3 1 D equal to the inner angle at C, and the angle 4 1 E equal to the inner angle at B; draw DE: DE 1 is the triangle required.

PROB. LVI. — *About a given circle to describe a triangle similar to a given triangle ABC.*

Find the centre of the circle, and draw any radius 1 2; produce the base of the triangle to 3 and 4, and at point 1 make the angle 2 1 6 equal to the external angle AB 3; also the angle 2 1 5, on the other side of the line 2 1, equal to the external angle AC 4; at points 2 5 6 draw tangents to the circle by erecting perpendiculars to the radii, cutting in DEF: DEF is the required triangle.

THE MEASUREMENT OF ANGLES. — For the measurement of angles, the circumference of the circle is divided into 360 equal parts, called degrees. The sign of a degree is thus, $^{\circ}$. If two diameters of the circle be drawn perpendicular to each other, they will divide the circle into four equal parts, each of which will contain the fourth of 360 degrees; that is, 90 degrees. As these lines are at right angles to each other, it follows, therefore, that a right angle contains 90 degrees. An obtuse angle will contain more than 90° , because it is greater than a right angle, and an acute angle less than 90° , because it is less than a right angle.

PROB. LVII. — *To construct an angle of 45° at point B on a given line AB.*

By Prob. 5, erect a perpendicular at point B. ABC will therefore be a right angle; that is, an angle of 90° . Bisect the right angle, and draw the line DB: DBA is an angle of 45° . If the right angle is trisected, it will give an angle of 30° and 60° .

PROB. LVIII. — *At a point C in the line AB, to make an angle of 60° , 45° , 30° , or 15° .*

With centre C, any radius, describe an arc cutting AB in I; with centre I, same radius, cut the arc in D; draw DC: it will make an angle of 60° with AB. Bisect angle DCA by line EC: EC will make an angle of 30° with AB. Bisect angle ECA by line FC: FC will make an angle of 15° with BC. Bisect angle ECD by line GC: GC will make an angle of 45° with AB.

PROB. LIX. — *To construct an angle of 1° , 2° , 3° , 4° , 5° .*

Construct an angle of 60° by Prob. 58, ABC; bisect it by DB; divide DC on the arc with the compasses into three equal parts; divide the lower division into two equal parts, and divide the lower half of this into five equal parts: these will respectively equal 1° , 2° , 3° , 4° , and 5° .

The three angles of a triangle added together contain 180° , which is equal to two right angles. If, therefore, the number of degrees in two of the angles are given, the third angle may be known in this manner: (Suppose angle B contains 90° , A 60° , which together will make 150° , the third or C will contain 30° , and the whole will equal 180° .) If the three angles are equal, as in the equilateral triangle, then, as the three together equal 180° , each angle must equal one-third of 180° , or 60° . In an isosceles triangle, if the number of degrees in an angle at the base be given, and it be required to find the value in degrees of the vertical angle, multiply the number of degrees in one angle at the base by 2, and the difference between the product and 180° will equal the number of degrees in the vertical angle.

PROB. LX. — *On a given base AB, to construct a triangle having angles of 60° , 30° , and 90° .*

At B construct a right angle, — that is, erect a perpendicular; at A make an angle of 60° by Prob. 58; continue the line to cut the perpendicular erected at B, in point C: ABC is the required triangle.

PROB. LXI. — *To construct an isosceles triangle on a given base AB, having a vertical of 90° .*

The two angles at the base of an isosceles triangle are equal; and, as the vertical angle is to be 90° , each angle at the base will be half 90° , or 45° . At point A make a right angle; bisect it, thereby making angle CAB equal to 45° ; at B make an angle equal to CAB: ABC will be the triangle required.

PROB. LXII. — *To draw a line from a given point A, to cut a line BC in an angle having a required number of degrees.*

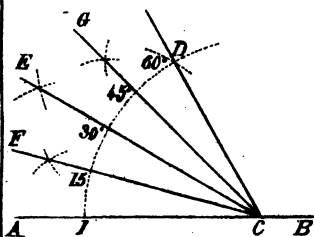
Let the angle required be 60° . Draw a line AD through A, parallel with line BC. On AD, at the point A, make an angle of 60° , producing the line to cut BC in E: AE is the required line. If the angle required be 30° , bisect angle DAE in line AF: line AF produced will make an angle of 30° with BC.

If the angle required be 90° , drop a perpendicular from the given point, as I, to cut the given line 23 in 5: 15 will make 90° with 23. If the angle required be 45° , bisect the angle 4 15 by line 16; then 16 will make an angle of 45° with line 23.

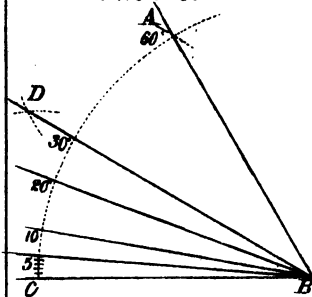
PROB. LXIII. — *To place two given lines AB and CD at right angles at their centres.*

Draw a line AB, equal to the given line AB; bisect it in O by a perpendicular. Bisect CD; set off half of CD on the line bisecting AB in CD: AB and CD will be equal to AB and CD, and be placed at right angles at their centres.

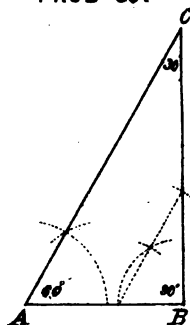
PROB 58.



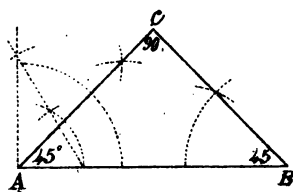
PROB 59.



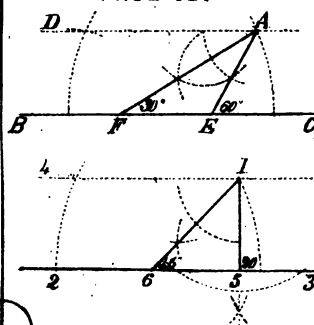
PROB 60.



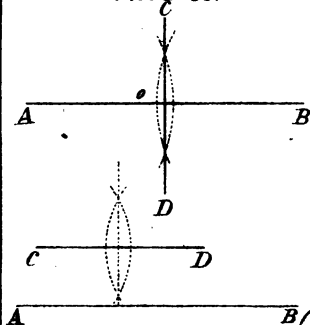
PROB 61.



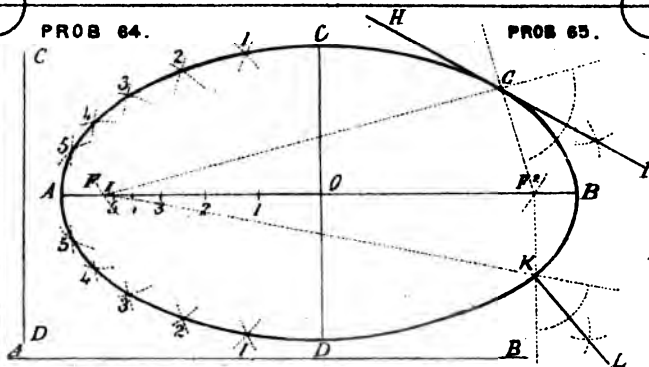
PROB 62.



PROB 63.

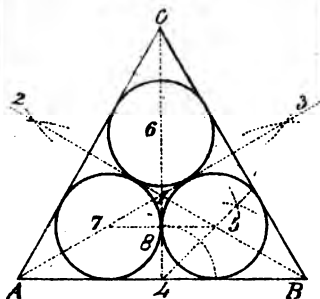


PROB 64.

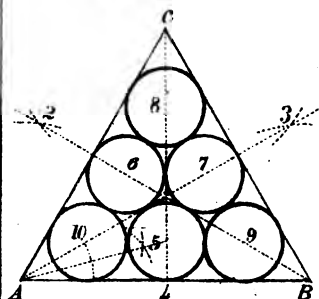


PROB 65.

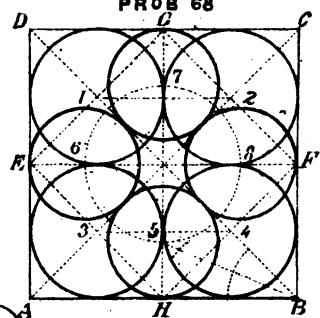
PROB 66.



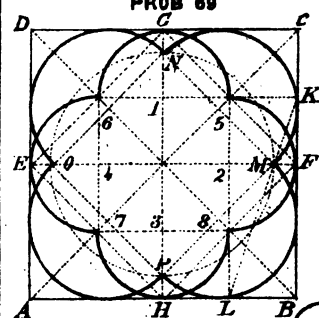
PROB 67.



PROB 68.



PROB 69.



PROB. LXIV. — *To construct an ellipse, its diameters AB and CD being given.*

Place AB and CD at right angles at their centres. Find the foci of the ellipse by taking half AB as radius, and C as centre, cutting AB in F1 and F2. Divide the distance from O to F1 in parts decreasing towards F1; figure them 1, 2, 3, 4, 5. With radii A1, A2, A3, A4, A5, from centre F1, describe arcs between CA and DA. With radii B1, B2, B3, B4, B5, centre F2, intersect the arcs already struck in points 1', 2', 3', 4', 5'. Draw the curve of the ellipse through the intersections. Repeat the process on the opposite side of CD for the other half of the ellipse. The student should remember that the arc struck from F1, with radius A1, will be cut by that struck from F2, with radius B1; the arc of radius A2 will be cut by the arc of radius B2, and so on.

PROB. LXV. — *At a given point in the curve of an ellipse, to draw a tangent or a perpendicular to the curve.*

Let the point G be where the tangent is required. From F1 draw a line through G, and from F2 a line to G. Bisect the angle thus obtained, and the bisecting line H1 will be the tangent. Let K be the point where the perpendicular is required. From F1 and F2 draw lines through K. Bisect the angle thus obtained, and the bisecting line KL will be the required perpendicular.

PROB. LXVI. — *Within an equilateral triangle ABC, to inscribe three equal circles, touching each other and two sides of the triangle.*

From A and C, describe arcs intersecting in 2 and 3. Draw A3 and B2, cutting in point 1 the centre of the triangle. Draw C4. Bisect angle 1 4 B, the bisecting line cutting B2 in 5. Set off the distance 1 5 from 1 on lines C4 and A3 in points 6 and 7. Then 5, 6, 7 will be the three centres of the required circles. Draw line 5 7, and 5 8 will be the radius of the required circles.

PROB. LXVII. — *Within an equilateral triangle ABC, to inscribe three equal circles, touching each other and one side of the triangle each.*

Proceed as in the last problem to find the centre of the triangle, and to draw lines from each angle to the opposite side. Bisect the angle 1 A4, producing the bisecting line to cut C4 in point 5. Set off the distance 1 5 from 1 on lines A3 and B2 in points 6 and 7. Then 5, 6, 7 will be the centres, and 5 4 the radius of the required circles.

If it be required to inscribe three other similar circles, each touching two others, and two sides of the triangle, from centre 5, radius 5 6, cut the lines A3 and B2 in 9 and 10; from centre 6, same radius, cut C4 in 8. Then 8, 9, 10 will be the centres of the three additional circles.

PROB. LXVIII. — *Is a simple geometrical exercise.*

PROB. LXIX. — *Within a given square to inscribe four semicircles, having adjacent diameters, (1) touching two sides of the square, and (2) touching one side each.*

Draw the diagonals and diameters of the square AC, BD, EF, GH, and the second diagonals of the smaller squares EG, EH, FG, FH. Draw lines joining the centres of the smaller square, thereby obtaining points 1, 2, 3, 4, the centres of the semicircles touching one side of the square 1 G being the radius.

For the semicircle touching both sides of the square, bisect CF and HB in points K and L, by producing the lines through 1 and 2. Draw KL cutting EF in M. With the centre of the square as centre, to M as radius, describe a circle to cut the diameters in N, O, P. Join these points by lines cutting the diagonals in 5, 6, 7, 8, the centres of the semicircles touching two sides of the square, 5 M being the radius.

PROB. LXX. — *To draw a tangent to an arc of a circle at a given point A, when the centre cannot be used.*

Draw any chord AB, and bisect it by the perpendicular CD. Draw DA, and make angle DAE equal to DAC. Produce the line EA: EF is the required tangent.

PROB. LXXI. — *To describe a circle having a given radius AB, tangential to a given line CD, and touching a given circle; the given line and circle being at a distance from each other less than the diameter of the required circle.*

Draw a line 1 2 parallel to CD, at the distance of AB from it. Draw a radius EH of the given circle, and produce it to 3, making H3 equal AB. With centre E, radius E3, describe an arc to cut 1 2 in 4: 4 is the centre of the required circle.

PROB. LXXII. — *Within a given triangle ABC, to inscribe an oblong, having a given side DE.*

From B set off B1 equal to DE, on the side BC. From 1 draw a line parallel to AB, to cut AC in F. Mark off BG equal to 1F on the line BA. Draw FG. From G let fall a perpendicular GH to BC. From H mark off line GF on BC in L. Draw FL. GHFL is the required oblong.

PROB. LXXIII. — *To inscribe a square in a given trapezium, which has adjacent pairs of sides equal, ABCD.*

Draw a diagonal AB, bisecting the trapezium and the angle at A. Find the centre of the figure in point 1, by bisecting another angle, as at C. At 1 erect a perpendicular to AB, as 1 2, and bisect the right angles on either side of 1 2, producing the bisecting lines to cut the trapezium in EFGH. Draw EF, FG, GH, HE, which will form the required square.

PROB. LXXIV. — *To divide a circle into any number of parts, which shall be equal both in area and outline.*

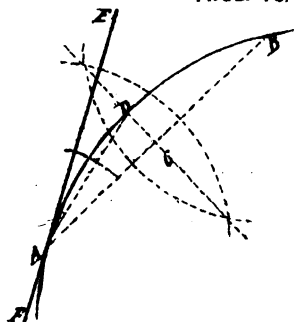
Draw any diameter 1 2. Divide it into the required number of parts — four, for example — in the points 1, 3, 4, 5. Bisect 1 3, and strike a semicircle on it above the line 1 3, and on 5 2 a similar one below the line. Bisect the remaining part 3 2 of the line 1 2, and describe a semicircle below the line, and a similar one on 1 5 above it. With 3 and 5 as centres, describe semicircles above and below the line 1 2 respectively, completing the figure.

PROB. LXXV. — *To construct any regular figure, from a three-sided to a twelve-sided, on a given base AB.*

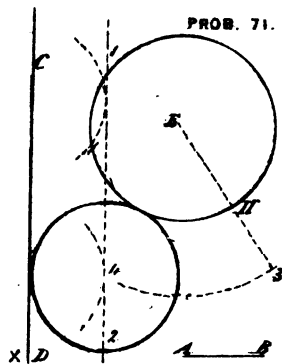
Bisect AB by a perpendicular, and with centre A, radius AB, describe an arc to cut the perpendicular in C. Divide the arc CB into six equal parts. C is the centre and CA the radius of a circle in which a hexagon may be inscribed on AB, as above. For a pentagon, mark off the distance C5 on the perpendicular in 5'; and with 5' as centre, 5'A radius, describe the circumscribing circle. For a nonagon, mark off the distance from C to 3 above C on the perpendicular, and for the duodecagon mark off the distance from C to B, for the centres of the circumscribing circles.

It will be observed, that, for a hexagon, C is the centre of a circumscribing circle, and that, for every side beyond six in a required figure, one division of CB is marked above C on the perpendicular, to find the centre of the circle. Thus 7, 8, 9, 10, 11, 12, are the centres of circumscribing circles for polygons having a corresponding number of sides.

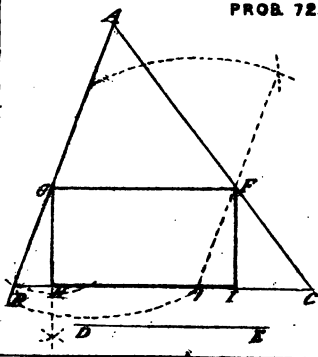
PROB. 70.



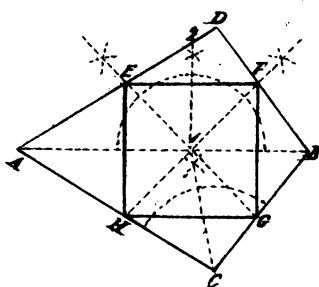
PROB. 71.



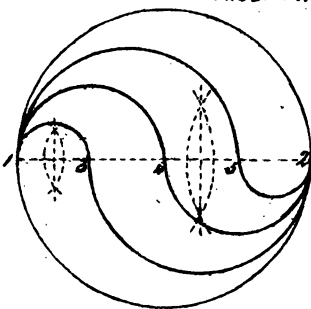
PROB. 72.



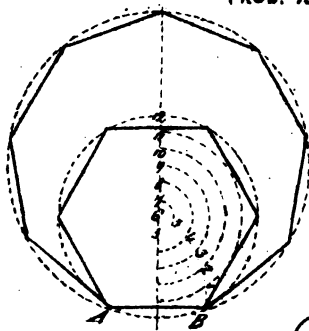
PROB. 73.



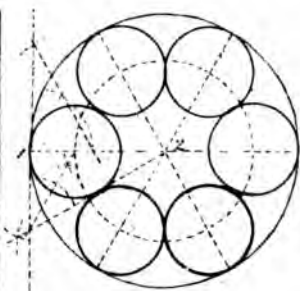
PROB. 74.



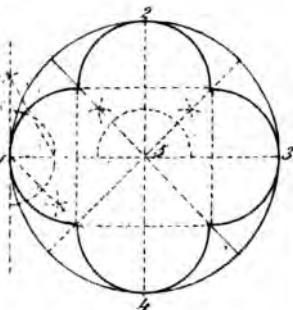
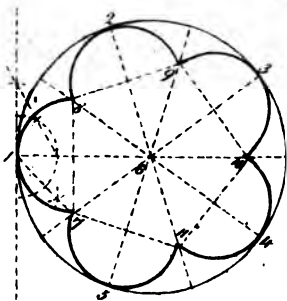
PROB. 75.



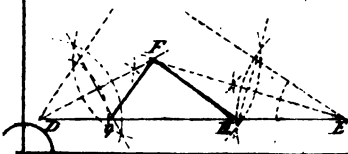
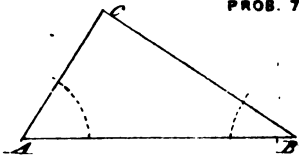
PROB. 76



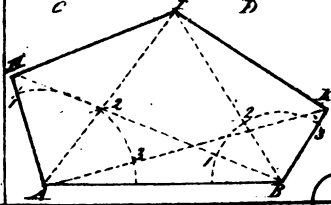
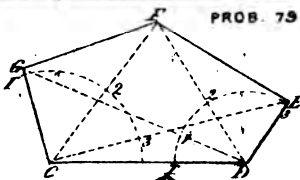
PROB. 77



PROB. 78



PROB. 79



PROB. LXXVI. — *Within a given circle to inscribe any number of equal circles.*

If the number of inscribed circles is uneven, — as five, for example, — divide the circumference of the circle into the same number of parts as are required circles to be inscribed, as in Prob. 38, in points 1, 2, 3, 4, 5; draw lines from them to the centre 6; draw a tangent to the circle at any one of these points, as 2; bisect the angle at the centre of the circle, on either side of line 2 6, by producing line 4 6, to cut the tangent in 7; bisect angle 2 7 6, producing the line to cut 2 6 in 8: 8 is the centre, and 8 2 the radius, of one of the circles to be inscribed. With centre 6, radius 6 8, describe a circle to find the other centres on lines 1, 3, 4, 5; and with these centres, radius 8 2, describe the required circles.

If the required number of inscribed circles be even, as four, six, eight, &c.

Let the example be to inscribe six circles: divide the circumference, and draw lines to the centre from the divisions, as before; also draw the tangent to one line, as at point 1, in line 1 2; bisect the angle on one side of 1 2 at the centre by Prob. 7, and proceed as before.

PROB. LXXVII. — *Within a given circle to inscribe any number of equal semicircles with adjacent diameters.*

Divide the circumference of the circle into the number of parts as are required semicircles to be inscribed, as in the last problem, in points 1, 2, 3, 4, 5; from these points draw diameters; from any point, as 1, draw a line at the angle of 45° to the diameter from 1 to cut the adjacent diameter in point 7, by erecting a perpendicular, and bisecting the right angle; set off the distance 6 7 on each alternate semidiameter from 7, in points 8, 9, 10, 11; draw 7 8, 8 9, &c., and these lines will form the adjacent diameters of the required semicircles. Where these diameters intersect the diameters of the large circle will be the centres of the semicircles; and the distance from the intersections to 1, 2, &c., will be their radii. This rule applies to all uneven numbers of required semicircles. If the number of inscribed semicircles is required to be even, as four, six, eight, draw the diameters from the divisions 1, 2, 3, &c., as before, and bisect the angles at the centre of the circle, to form other diameters, as shown in the diagram, and proceed as with the method for uneven numbers.

PROB. LXXVIII. — *To construct a triangle whose angles shall be equal to those of a given triangle ABC, and whose periphery or outline shall be equal to a given line DE.*

At D and E make angles equal to those at A and B respectively; bisect the angles at D and E, producing the lines to meet in F: bisect lines DF, EF, letting the bisecting lines cut DE in GH; draw FG and FH: FGH is the required triangle.

PROB. LXXIX. — *On a given line AB to construct a figure similar to a given figure CDEFG.*

Draw lines from C and D to angles at G, F, and E; at A and B make angles equal to those at C and D, figuring them 1, 2, and 3, as shown in diagram; where the line from A through 1 meets the line from B through 1, and A through 2 meets the line from B through 2, &c., in points H, I, and K, will be the angles of the required figures. Draw HI, HK, to complete the figure: ABKIH will be similar to ODEFG.

ELLIPSES.

PROB. LXXX. — *To construct an ellipse, its major or transverse diameter AB, and its minor or conjugate diameter CD, being given.*

Place AB and CD at right angles at their centres, as shown; describe circles on both diameters, and trisect two adjacent right angles of the large circle, producing the trisecting lines to the opposite side of the circle, and figuring them 1, 2, both on the larger and smaller circles, starting from the larger diameter in all cases; join points 1 1 and 2 2 on the large circle, and project lines through 1' 1' and 2' 2' on the smaller circle; where these lines through similar figures, as 1, 1', intersect, will give points through which to draw the ellipse.

Same Problem, second method (by intersecting lines).

Place the two diameters AB and CD at right angles at their centres, and on them construct a rectangle; divide the semi-major diameter AE into any number of equal parts, — four, for example, — and the semicircle of the rectangle from A, as AI, into the same number of equal parts; starting from A, figure these divisions 1, 2, 3, and 1' 2' 3'; draw lines from D, one end of the minor diameter through 1, 2, 3, and from C the other end of the minor diameter, to 1' 2' 3': where these lines intersect will give points through which to draw the ellipse. Repeat the process in the other quarters.

Same Problem, third method (by intersecting arcs). See Prob. 64.

Same Problem, fourth method.

This method is by the use of pins and string. Place the diameters, and find the foci as in the last method; fix three pins at E, F, and C, and tie a piece of string to E and F, so that, when tightly strained, it shall make two straight lines EC and FC; remove the pin at C, and with a sharp pencil from C, with the string tightly strained, make the point traverse through ABC and D: this will form the required ellipse.

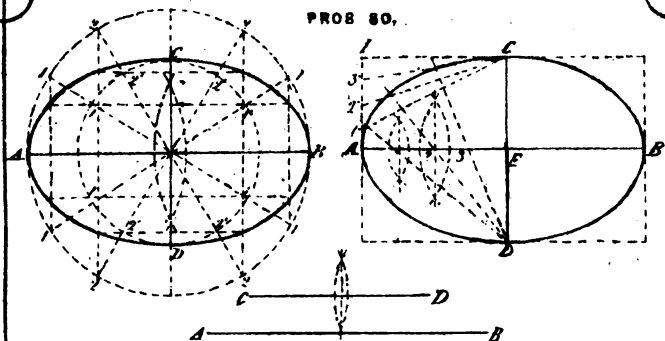
PROB. LXXXI. — *To find the centres and axes of a given ellipse.*

Draw any two parallel chords, and bisect them; draw a line through the bisection, terminated both ways by the circumference, as 1 2; bisect it in A: A is the centre of the ellipse. With centre A, any radius, cut the ellipse in three places, 3, 4, 5; draw 3 4, 4 5; bisect lines 3 4, 4 5, and draw the bisecting lines to terminate in the circumference of the ellipse in BC and DE: then BC and DE are the two diameters.

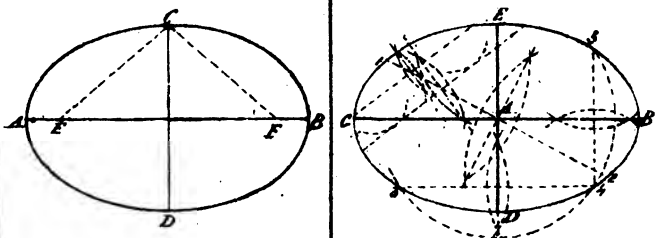
PROB. LXXXII. — *To describe an ellipse about a given rectangle, ABCD.*

Draw the major and minor axes indefinitely. Assume the foci 1 and 2, each equidistant from the centre of the ellipse. Place 1A and A2 in the same straight line, and bisect it. Place half the line on either side of O, on the line through 1 2, in E and F. Then E and F will be one diameter. With OE radius, centre 1, intersect the line which is at right angles to EF through O, in G and H. Then GH will be the other diameter of the ellipse. To complete the ellipse, proceed as in the third method, before described.

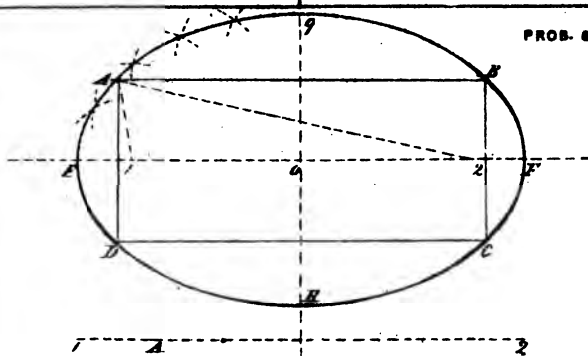
PROB. 80.



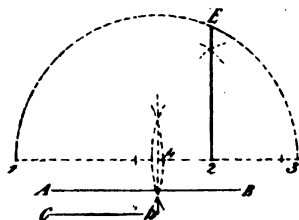
PROB. 81.



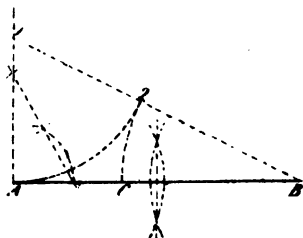
PROB. 82.



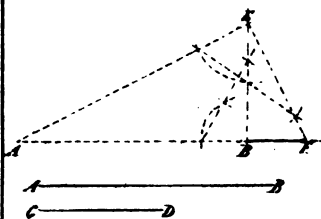
PROB. 83



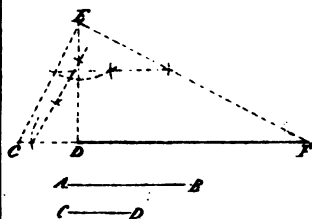
PROB. 84.



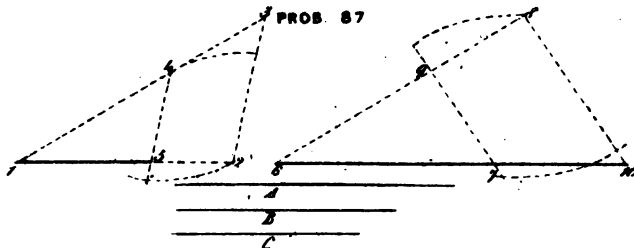
PROB. 85



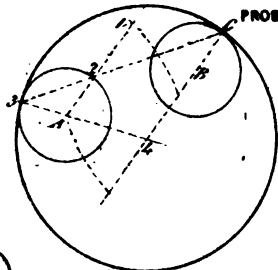
PROB. 86.



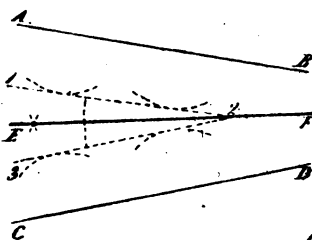
PROB. 87



PROB. 88



PROB. 89



PROPORTIONALS.

PROB. LXXXIII. — *To find a mean proportional to two given lines AB and CD.*

Make 1 2 equal AB, and 2 3 equal CD, in the same straight line 1 3; bisect 1 3 in point 4; with centre 4, radius 4 3, describe a semicircle on 1 3; at 2 erect a perpendicular to 1 3, intersecting the semicircle in E: 2E is the mean proportional required.

PROB. LXXXIV. — *To divide a given line AB into extreme and mean proportion.*

At A erect a perpendicular A1 equal to half AB; draw 1B. With centre 1, radius 1A, describe an arc to cut 1B in 2; with centre B, radius B2, describe an arc to cut AB in C: AC is the extreme, and BC the mean, proportion required; i.e., AC is as much smaller than CB as CB is than AB; in other words, AC is to CB as CB is to AB.

PROB. LXXXV. — *To find a third proportional less to two given lines AB and CD.*

Draw BE perpendicular to AB, equal to CD, the shorter of the given lines; draw AE; produce AB indefinitely beyond B; at E draw a line perpendicular to AE to cut AB produced in F: BF will be the third proportional less.

PROB. LXXXVI. — *To find a third proportional greater to two given lines AB and CD.*

Draw DE (the longer of the two given lines) perpendicular to CD, and equal to AB; join EC; produce CD beyond D; at E draw a line perpendicular to EC to cut CD produced in F: DF is the third proportional greater.

NOTE. — It will be well for the student to notice that these problems will be easily remembered by one feature. If the third proportional required be *less*, the *shorter* of the two lines is to be made perpendicular to the longer, as though it were the altitude of a right-angled triangle, and the longer line were the base. If the third proportional required be *greater*, the *longer* line must be treated as the altitude, and the shorter as the base.

PROB. LXXXVII. — *To find a fourth proportional to three given lines AB and C.*

When the fourth proportional is to be *less*, draw 1 2 equal to B, and 1 3 equal to A, at any angle with 1 2; join 2 3; mark off 1 4 equal to C on the *longer* line 1 3; and from 4 draw a line 4 5 parallel to 2 3, to cut 1 2 in 5: then 1 5 will be the fourth proportional less.

When the fourth proportional is to be *greater*, draw 6 7 equal to B, and 6 8 equal to A, at any angle; set off 6 9 equal to C on line 6 8; draw line 9 7; from 8 draw a line parallel with 9 7 until it cuts 6 7 produced in 10; then 6 10 will be the fourth proportional greater.

PROB. LXXXVIII. — *To draw a circle touching two given circles, A and B, and one of them in a given point C.*

Draw a radius of the circle on which C is placed from C, and produce it; from A draw a radius parallel with the radius from B, to cut circle A in 2; draw C2 and produce it to cut circle A in 3; from 3 draw a radius to cut the produced radius from C in 4: 4 is the centre, and 4C the radius, of the required circle.

PROB. LXXXIX. — *To bisect the angle formed by two converging lines, when the angle is not available.*

Draw any two lines 1 2 and 3 2 parallel to the two given lines AB and CD, and at the same distance from each, to cut in point 2; bisect angle 1 2 3, and the bisecting line EF would bisect the angle made by the two given lines AB and CD.

AREA OF FIGURES.

THE area of a triangle is measured by multiplying its base by its altitude, and dividing by 2: thus, suppose the base of a triangle be equal to 4 inches, and its altitude be equal to 3 inches, then $4 \times 3 = 12 \div 2 = 6$; the area of the triangle will be equal to 6 inches: the area of a parallelogram is obtained by multiplying its base and altitude together; thus, a parallelogram having a base of $4\frac{1}{2}$ inches, and an altitude of 3 inches, then $4\frac{1}{2} \times 3 = 13\frac{1}{2}$ inches, which will be the area of the parallelogram.

AXIOM I. — *Triangles on the same base, or on equal bases, and between the same parallels, are equal in area.*

The triangles on the base AB, and between the same parallels 1 2 and 3 4, are equal; i e, ABC, ABD, ABE, ABF, having the same base AB, and, through being between the same parallels, having the same altitude, are equal to one another in area.

AXIOM II. — *Parallelograms on the same base, or on equal bases, and between the same parallels, are equal in area.*

Thus the parallelograms ABCD, ABCE, ABEF, being between the same parallels 1 2, 3 4 and on the same base AB, are equal in area.

AXIOM III. — *A triangle on the same base, and between the same parallels as a parallelogram, is equal to half its area.*

Thus the triangle ABC equals half the area of a parallelogram ABCD.

NOTE. — The term, *between the same parallels*, is equivalent to saying, *having the same altitude.*

AXIOM IV. — *Triangles having the same or equal bases have to each other the ratio that their altitudes have, or, having the same or equal altitudes, the ratio that their bases have.*

Thus triangle ABC is only half the area ABD, and one-third of ABE, all being on the same base AB, and the altitude of ABC being only one-half of ABD and one-third of ABE.

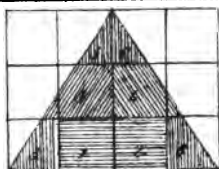
Again: the triangle FIH is twice as great as FGH, because, though having the same altitude, it has double the base; for the same reason FKH is twice and a half FGH, because FK, the base, is twice and a half FG, the altitudes of both being the same.

PROB. XC. — *To construct an oblong having four times the area of a given triangle ABC.*

Find the altitude AD of the triangle; make EF equal the base BC of the triangle; erect a perpendicular EH at E, twice the height of the altitude AD, and complete the oblong required.

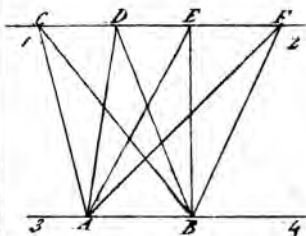
PROB. XCI. — *To draw a triangle equal in area to a given trapezium ABCD.*

Produce the base AB beyond A; draw a diagonal CA; from D draw a line parallel with CA, to cut BA produced in E; draw CE: CEB will be equal in area to the trapezium ABCD.

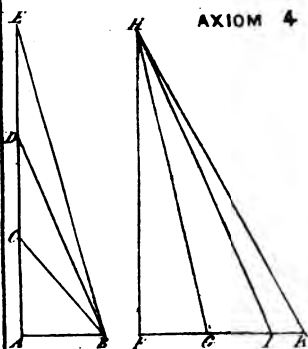


1	2	3	4	13
5	6	7	8	
9	10	11	12	2

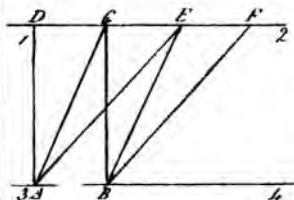
AXIOM 1.



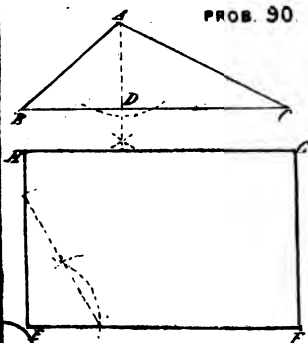
AXIOM 4



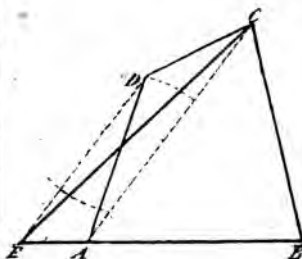
AXIOMS 2 & 3.



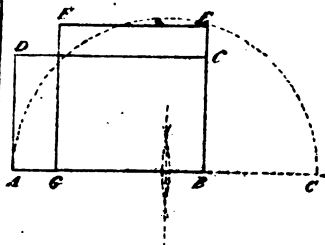
PROB. 90.



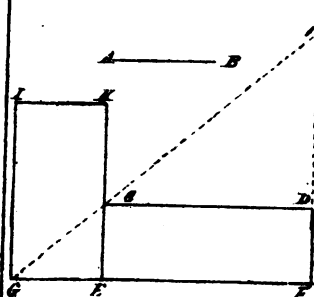
PROB. 91.



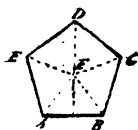
PROB. 32.



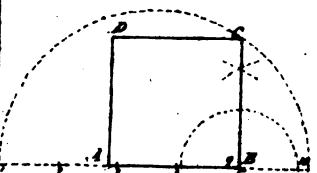
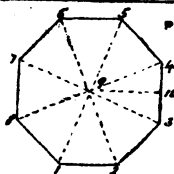
PROB. 33.



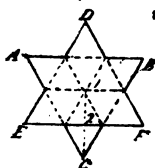
PROB. 34.



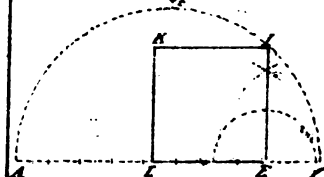
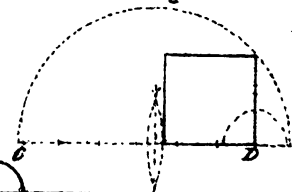
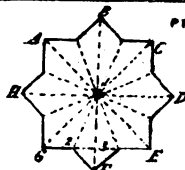
PROB. 35.



PROB. 36.



PROB. 37.



PROB. XCII. — *To construct a square equal in area to a given oblong ABCD.*

Find a mean proportional between two adjacent sides of the oblong AB and BC, as shown in the diagram, by Prob. 83: the mean proportional will be equal to one side of the square. Construct the square BEFG.

PROB. XCIII. — *To construct a parallelogram on a given base AB, equal in area to a given parallelogram CDEF.*

Produce the base EF to G, making EG equal to AB; produce FD indefinitely, and draw a line from G through C to cut it in I; construct a parallelogram of the two lines AB and DI, GEKI, which will be equal in area CDEF.

PROB. XCIV. — *To construct a triangle equal in area to any given regular polygon ABCDE.*

A regular polygon may be divided into as many triangles, having equal bases and altitudes, as the polygon has sides. By Axiom 1, triangles on equal bases, and having equal altitudes, are equal in area: therefore all the triangles forming the polygon are equal to one another in area. Divide the given polygon into five triangles by drawing lines from the angles to the centre F; set off the length of the bases in one straight line AA'; and on one base AB construct a triangle equal to one of the triangles forming the polygon AB, F; join F·A': triangle AF·A' will be equal to ABCDE, the given polygon.

PROB. XCV. — *To construct a square equal in area to any regular polygon 1 2 3 4 5 6 7 8.*

By Prob. 83, find a mean proportional between half the periphery or circumference of the polygon, as 1 2 3 4 5, and half the diameter, as 9, 10: the mean proportional will be equal to one side of the square. Construct the square ABCD, which will equal the polygon.

NOTE. — If the polygon have an uneven number of sides, — as seven, for example, — a mean proportional must be found between half the periphery of the polygon as before, and the altitude of one of the equal triangles into which the polygon may be divided.

PROB. XCVI. — *To construct a square equal in area to a given regular figure ABCDEF.*

Resolve the figure into twelve equilateral triangles, as shown in the diagram; find a mean proportional between half the periphery or outline of the figure CFBD and the altitude of one of the equilateral triangles composing it, as 1 2. On the mean proportional construct the square.

PROB. XCVII. — *To construct a square equal in area to a given figure ABCDEFGH.*

Resolve the figure into equal triangles, and find the altitude of one of them, as G I 2, which will be 1 3; find a mean proportional between half the periphery or outline ABCDE and 1 3, the altitude of one triangle: on the mean proportional construct the required square, EIKL. [This rule may be followed whenever a figure can be resolved into equal triangles.]

PROB. XCVIII. — *To construct a parallelogram on a given base AB, equal in area to a given square CDEF.*

Find a third proportional less to the given base AB, and one side of the given square, as CD, by Prob. 85: the third proportional will equal the altitude of the required parallelogram ABGH. [This is simply a reversal of the Prob. 92, to make a square equal to a given parallelogram. In that problem, the two extremes are given in the sides of the oblong; and the mean, which will be the side of the square, is required. In this problem, the two longer lines—i.e., the base of the parallelogram, and side of the square—are given; and it is required to obtain the extreme proportion, or third proportional less, for the altitude of the parallelogram; but if the base of the required parallelogram be less than the side of the given square, then it will be necessary to find a third proportional greater, for the altitude of the parallelogram.]

PROB. XCIX. — *To construct a square equal in area to a given trapezium ABCD.*

Resolve the trapezium into a triangle of equal area ADE, by Prob. 91, and the triangle into a parallelogram of equal area AEFG; find a mean proportional between the base AE and the altitude AG of the parallelogram, and on it construct the required square AHIK.

PROB. C. — *To reduce or increase the number of sides of a regular figure without changing the area.*

Suppose it is required to reduce a regular pentagon ABCDE to a trapezium of equal area: produce the side BA indefinitely beyond A; draw a diagonal DA cutting off one angle, as DEA; from E draw a line parallel with the diagonal AD, to cut the produced side in F; draw DF: then the trapezium FBOD will equal the pentagon ABCDE.

If it be required to increase the number of sides of a regular figure, without changing its area, the process is reversed: let the figure to be increased be a hexagon ABCDEF, to be made into a heptagon; assume any point G in a side AB; draw a line from G to the next angle, as F; from A draw a line AH parallel with FG; assume any point I in the line AH, and draw FI, GI; then GBCDEFI will be a heptagon of equal area to the given hexagon. In like manner, by repeating the process, the heptagon could be increased to an octagon of equal area, as shown at the angle B.

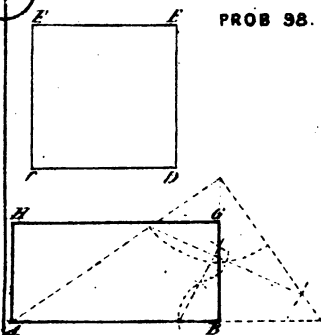
PROB. CI. — *To construct a square, circle, or regular polygon, equal in area to two given similar figures.*

Let AB and CD be the sides of two given squares, or the diameters of two circles or regular polygons; place them at right angles, as EF, FG; draw GE: GE will be the side of a square, or the diameter of a circle or regular polygon whose area shall be equal to the two similar figures constructed on AB and CD.

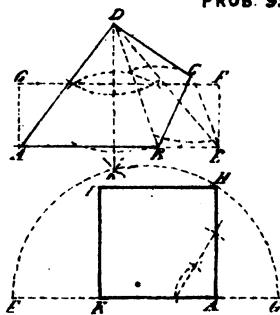
PROB. CII. — *To find the difference in area between two given squares.*

Let AB and CD represent the sides of two given squares; draw EF equal to the shorter line CD; at its extremity, F, erect a perpendicular indefinitely, as FG; with centre E, radius AB, intersect FG in H: then a square constructed on HF will represent the difference between two squares whose sides are equal to A, B, C, and D.

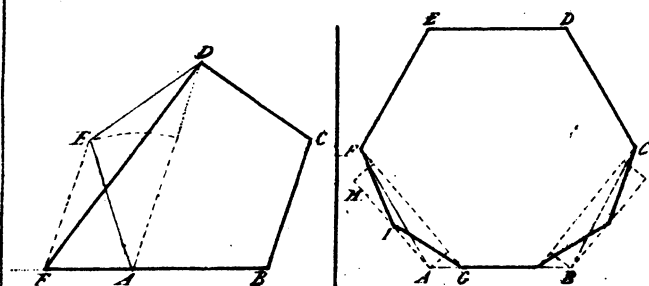
PROB. 98.



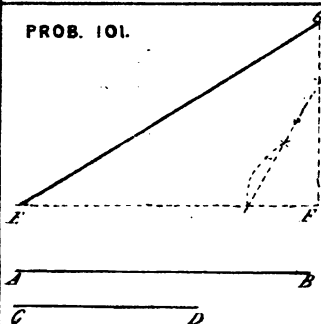
PROB. 99.



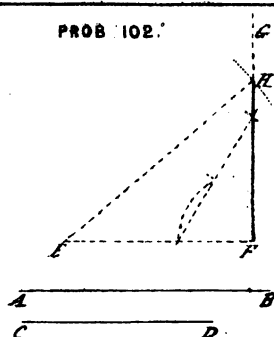
PROB. 100.



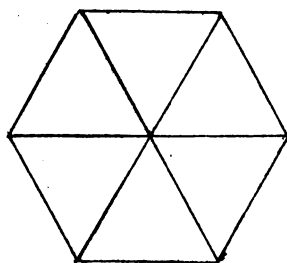
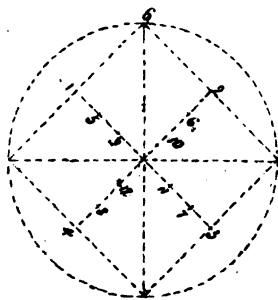
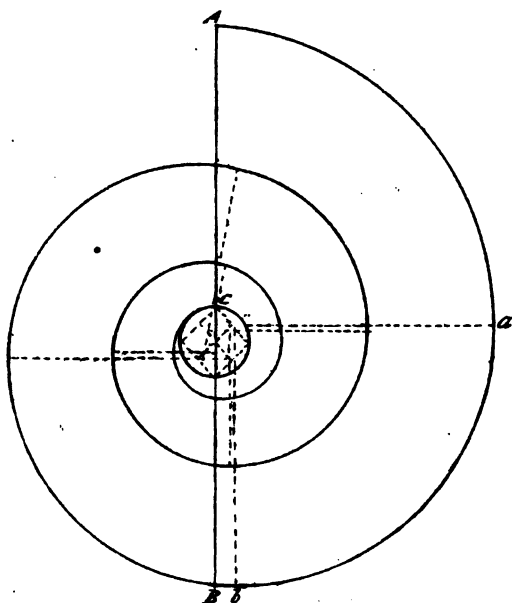
PROB. 101.



PROB. 102.



PROB 103



PROB. CIII. — *To construct a spiral curve, its greatest diameter being given.*

Let AB be the given diameter; bisect in C; divide BC into four equal parts; on the part next the centre C as diameter, describe a circle which will be the eye of the spiral; within the circle inscribe a square, one of its diagonals coinciding with a portion of line AB; draw the diameters of the square 1 3, 2 4; divide each diameter into six equal parts, and, starting from the next division on the diagonal 1 3 nearest 1, number them 5, 6, 7, 8, 9, 10, 11, 12: produce 1 2 indefinitely; and with centre 1, radius 1A, describe an arc to cut the produced line in *a*; produce 2 3 indefinitely, and with radius 2A describe an arc to cut the produced line 2 3 in *b*; complete the spiral in this manner, remembering that each arc forming it is terminated by a line drawn from its centre through the next highest figure numerically.

To calculate the number of degrees in the angle of a regular polygon.

The rule is, that the sum of the internal angles of a polygon is equal to twice as many right angles as the figure has sides, minus four angles.

The reason of this is, that a regular polygon has the same number of equal triangles as sides. Each triangle contains two right angles. All the angles meeting in a point (the centre of the polygon) will equal four right angles, which, subtracted from twice as many right angles as the figure has sides, leaves the gross number of degrees in the angles. A division of this sum by the number of angles gives the number of degrees in each angle.

Thus, a hexagon has six sides, which, multiplied by 2, gives 12; subtract 4 will leave 8 right angles, which, multiplied by 90, gives 720°. Dividing these 720° by 6 (the number of angles) will give 120° as the angles of a hexagon. The following is a tabular calculation of the angle of each polygon up to a twelve-sided figure:—

Pentagon,	$5 \times 2 = 10 - 4 = 6 \times 90 = 540 \div 5 = 108^\circ$
Hexagon,	$6 \times 2 = 12 - 4 = 8 \times 90 = 720 \div 6 = 120^\circ$
Heptagon,	$7 \times 2 = 14 - 4 = 10 \times 90 = 900 \div 7 = 128\frac{1}{2}^\circ$
Octagon,	$8 \times 2 = 16 - 4 = 12 \times 90 = 1080 \div 8 = 135^\circ$
Nonagon,	$9 \times 2 = 18 - 4 = 14 \times 90 = 1260 \div 9 = 140^\circ$
Decagon,	$10 \times 2 = 20 - 4 = 16 \times 90 = 1440 \div 10 = 144^\circ$
Undecagon,	$11 \times 2 = 22 - 4 = 18 \times 90 = 1620 \div 11 = 147\frac{1}{11}^\circ$
Duodecagon,	$12 \times 2 = 24 - 4 = 20 \times 90 = 1800 \div 12 = 150^\circ$

ADDRESSED TO TEACHERS.

THE accuracy of Geometrical Drawing depends on the care used both in working with the compasses, describing arcs, and in the ruling of straight lines through points of intersection. Before commencing a lesson, the teacher should, therefore, see that the compass-pencils and drawing-pencils of the pupils are as sharply pointed as possible. The pencils used should be those marked *H* or *HH*. A ruler, with inches marked, and bevelled at the edge, will be found necessary for each pupil.

A page of this book will be sufficient for an hour's lesson. Each problem should be worked carefully on the black-board by the teacher, as large as possible, the result-lines being twice as thick as the conditional and working lines.

It will be found advisable to give alternate lessons as exercises of the memory; *i.e.*, instead of giving fresh problems, to write down the statements of problems given at the last lesson, and allow the pupils to work them from memory, without the assistance of diagrams. A class should be taken through the definitions at least twice before the problems are commenced.

Smooth copy-book paper, without lines, is cheapest and most suitable for working geometrical problems on.

The teacher should insist, from the first, that all points be figured as obtained, and every point used be distinguished by a separate letter or numeral.

The compasses must be held by the head, with the thumb and first two fingers, the legs being untouched by the fingers.

The pupils should be taught to bear very lightly on the pencil-leg of the compasses, and to strike circles or arcs from the fingers, and not from the wrist. The pencil should be held nearly upright in ruling lines; and its point should be like the point of a screw-driver, not conical, as for free-hand drawing: this applies also to the compass-pencil.

A page of the pupil's book in which the problems are worked should be divided thus

 before commencing worked in each division; the statement of the problem to be neatly written at the top of the space allotted to it.

INDEX TO DEFINITIONS AND PROBLEMS.

LINES AND ANGLES.

Definition.	Pages.
1 What is a point? . . .	2, 3
2 " is a straight line? . . .	2, 3
3 " is a curved line? . . .	2, 3
4 " are parallel lines? . . .	2, 3
5 " is an angle? . . .	2, 3
6 How many kinds of angles are there? . . .	2, 3
7 What is a right angle? . . .	2, 3
8 " is an obtuse angle? . . .	2, 3
9 " is an acute angle? . . .	2, 3

TRIANGLES.

10 What is a triangle? . . .	4, 5
What are those named from their angles? . . .	4, 5
What are those named from their sides? . . .	4, 5
11 What is a right-angled triangle? . . .	4, 5
12 " is an obtuse-angled triangle? . . .	4, 5
13 What is an acute-angled triangle? . . .	4, 5
14 What is an equilateral triangle? . . .	4, 5
15 " is an isosceles triangle? . . .	4, 5
16 " is a scalene triangle? . . .	4, 5
17 " are the three sides of a triangle usually called? . . .	4, 5
Which is the vertical angle? . . .	4, 5
Which are the angles at the base? . . .	4, 5
18 What is the altitude or height of a triangle? . . .	4, 5

QUADRILATERALS OR QUADRANGLES.

What is a quadrilateral or quadrangle? . . .	6, 7
How many quadrilaterals are there? . . .	6, 7
What is a parallelogram? . . .	6, 7
19 " is a square? . . .	6, 7
20 " is an oblong? . . .	6, 7
21 " is a rhombus? . . .	6, 7
22 " is a rhomboid? . . .	6, 7
23 " is a trapezium? . . .	6, 7
24 " is a trapezoid? . . .	6, 7

POLYGONS.

What is a polygon? . . .	6, 7
25 " is a regular polygon? . . .	6, 7
26 " is an irregular polygon? . . .	6, 7
27 Name all the polygons up to twelve sides . . .	6, 7

THE CIRCLE.

28 What is a circle? . . .	8, 9
" is its centre? . . .	8, 9
" is its circumference? . . .	8, 9
29 " is its diameter? . . .	8, 9
30 " is its radius? . . .	8, 9

THE CIRCLE, continued.

Definition.	Pages.
31 What is an arc and a chord? . . .	8, 9
32 " is a semicircle? . . .	8, 9
33 " is a quadrant? . . .	8, 9
34 " is a sector? . . .	8, 9
35 " is a tangent? . . .	8, 9
36 " is a diagonal? . . .	8, 9
37 " is a diameter? . . .	8, 9
38 When is a figure inscribed in another? . . .	8, 9
39 When is a figure inscribed in a circle? . . .	8, 9
40 When is a circle described about a figure? . . .	8, 9
41 When is a figure described about a circle? . . .	8, 9

PROBLEMS.

Problems.	
1 To bisect a given line AB, either straight or part of a circle . . .	10, 11
2 At a point C in a given line AB to draw a line perpendicular to it . . .	10, 11
3 To draw a line perpendicular to a given line AB from a point C outside and opposite to it . . .	10, 11
4 To draw a line perpendicular to a given line AB from a point C, quite or nearly over the extremity of AB . . .	10, 11
5 To erect a perpendicular at the extremity of a given line AB . . .	10, 11
6 To make an angle equal to a given angle A at a given point B in a line BC . . .	12, 13
7 To bisect a given angle ABC . . .	12, 13
8 To trisect a right angle ABC . . .	12, 13
9 To draw a line through a point C parallel to a given line AB . . .	12, 13
10 To draw a line parallel to a given line AB, at a distance from it equal to CD . . .	12, 13
11 To construct an oblong having two given sides AB and CD . . .	14, 15
12 To construct an oblong, its diagonal AB and one side CD being given . . .	14, 15
13 To construct a rhombus having a diagonal AB and a given side CD . . .	14, 15
14 To construct a rhombus having a given side AB and a given angle C . . .	14, 15
15 To construct a triangle of three given lines A, B, and C . . .	14, 15
16 To construct a triangle, its base AB and the angles at the base A and B being given . . .	14, 15
17 To divide a given line AB into any number of equal parts . . .	16, 17
18 To divide a line AB into the same proportions as a given divided line CD . . .	16, 17

Problems.	Pages.
19 To find the centre of a given triangle ABC	16, 17
20 To find the centre of a circle ABC	16, 17
21 To draw a tangent to a circle at a given point of contact A	16, 17
22 To draw a tangent to a circle from a point A outside the circumference	16, 17
23 Within a given triangle ABC to inscribe a circle	18, 19
24 About or outside a given triangle to describe a circle	18, 19
25 To make a trapezium equal to a given trapezium ABCD	18, 19
26 To make an irregular polygon equal to a given irregular polygon ABCDE	18, 19
27 To draw a line from a point C lying outside a line AB, to make an angle with line AB equal to a given angle D	18, 19
28 To draw a tangent to a given arc AB at a given point of contact C, when the centre of the arc cannot be based	18, 19
29 To construct a rhomboid, having its two sides AB and an angle given	20, 21
30 To construct a rhomboid, its two sides A and B, and its diagonal C, being given	20, 21
31 On a given base AB, to construct an equilateral triangle	20, 21
32 Within a given circle to inscribe an equilateral triangle	20, 21
33 On a given base AB to construct a square	20, 21
34 Within a given circle to inscribe a square	20, 21
35 On a given base AB to construct a regular hexagon	22, 23
36 Within a given circle to inscribe a regular hexagon	22, 23
37 On a given base AB to construct any regular polygon	22, 23
38 Within a given circle to inscribe any regular polygon	22, 23
39 On a given altitude AB to construct an equilateral triangle	22, 23
40 To construct an isosceles triangle having a given altitude AB and a given base CD	22, 23
41 About a given square ABCD to describe an equilateral triangle	24, 25
42 Within a given square ABCD to inscribe an equilateral triangle	24, 25
43 To construct an isosceles triangle on a given base AB and having a given vertical angle C	24, 25
44 Within a given square ABCD to inscribe an isosceles triangle having a given base EF	24, 25
45 To find the altitude of any triangle ABC	24, 25
46 To describe a circle through three given points ABC which are not to be in a straight line	24, 25
47 Within a given triangle ABC to inscribe a square	26, 27
48 Within a given regular hexagon to inscribe a square	26, 27
49 On a given diagonal AB to construct a square	26, 27
50 Within a given square ABCD to inscribe an octagon	26, 27

Problems.	Pages.
51 To inscribe a square in a given rhombus ABCD	26, 27
52 To inscribe a circle in a given rhombus ABCD	26, 27
53 About a given circle to describe an equilateral triangle	28, 29
54 About a given circle to describe a square	28, 29
55 Within a given circle to inscribe a triangle similar to a given triangle ABC	28, 29
56 About a given circle to describe a triangle similar to a given triangle ABC	28, 29
57 To construct an angle of 45°	28, 29
58 At a point C in the line AB to make an angle of 60° , 45° , 30° , or 15°	30, 31
59 To construct an angle of 1° , 2° , 3° , 4° , 5°	30, 31
60 On a given base AB to construct a triangle having angles of 60° , 30° , and 90°	30, 31
61 To construct an isosceles triangle on a given base AB having a vertical angle of 90°	30, 31
62 To draw a line from a given point A to cut a line BC in an angle having the required number of degrees	30, 31
63 To place two given lines AB and CD at right angles at their centres	30, 31
64 To construct an ellipse, its diameter AB and CD being given	32, 33
65 At a given point in the curve of an ellipse to draw a tangent or a perpendicular to the curve	32, 33
66 Within a given equilateral triangle to inscribe three circles touching each other and two sides of the triangle	32, 33
67 Within a given equilateral triangle to inscribe three circles touching each other and one side of the triangle	32, 33
68 Within a given square to inscribe four circles, one touching two sides of the square, and two touching one side	32, 33
69 Within a given square to inscribe four semicircles having adjacent diameters, one touching two sides of the square, and two touching one side	32, 33
70 To draw a tangent to an arc of a circle at a given point A when the centre cannot be used	2, 3
71 To describe a circle having a given radius AB, tangential to a given line CD, and touching a given circle; the given line and circle being at a distance from each other less than the diameter of the required circle	2, 3
72 Within a given triangle ABC, to inscribe an oblong having a given side DE	2, 3
73 To inscribe a square in a given trapezium, which has adjacent pairs of sides equal ABCD	2, 3
74 To divide a circle into any number of parts, which shall be equal both in area and outline	2, 3
75 To construct any regular figure from a three-sided to a twelve-sided, on a given base AB	2, 3

Problems.	Pages.
76 Within a given circle to inscribe any number of equal circles . . .	4, 5
77 Within a given circle to inscribe any number of equal semicircles, with adjacent diameters . .	4, 5
78 To construct a triangle whose angles shall be equal to those of a given triangle ABC, and whose periphery or outline shall be equal to a given line DE . . .	4, 5
79 On a given line AB to construct a figure similar to a given figure CDEFG . . .	4, 5

ELLIPSES.

80 To construct an ellipse, its major or transverse diameter AB and its minor or conjugate diameter CD being given . . .	6, 7
81 To find the centre and axes of a given ellipse . . .	6, 7
82 To describe an ellipse about a given rectangle ABCD . . .	6, 7

PROPORTIONALS.

83 To find a mean proportional to two given lines AB and CD . .	8, 9
84 To divide a given line AB into extreme and mean proportion, . .	8, 9
85 To find a third proportional less to two given lines AB and CD . . .	8, 9
86 To find a third proportional greater to two given lines AB and CD . . .	8, 9
87 To find a fourth proportional to three given lines A, B, and C . .	8, 9
88 To draw a circle touching two given circles, A and B and one of them in a given point C . . .	8, 9
89 To bisect the angle formed by two converging lines, when the angle is not available . . .	8, 9

AREA OF FIGURES.

Axioms.	
1 Triangles on the same base or on equal bases, and between the same parallels, are equal in area . . .	10, 11
2 Parallelograms on the same base or on equal bases, and between	

Axioms.	Pages.
the same parallels, are equal in area . . .	10, 11
3 A triangle, on the same base and between the same parallels as a parallelogram, is equal to half its area . . .	10, 11
4 Triangles having the same or equal bases have to each other the ratio that their altitudes have, or having the same or equal altitudes, the ratio that their bases have . . .	10, 11

PROBLEMS.

Problems.	
90 To construct an oblong having four times the area of a given triangle ABC . . .	10, 11
91 To draw a triangle equal in area to a given trapezium ABCD . . .	10, 11
92 To construct a square equal in area to a given oblong ABCD . . .	12, 13
93 To construct a parallelogram on a given base AB, equal in area to a given parallelogram CDEF . . .	12, 13
94 To construct a triangle equal in area to any given regular polygon ABCDE . . .	12, 13
95 To construct a square equal in area to any regular polygon 12345678 . . .	12, 13
96 To construct a square equal in area to a given regular figure ABCDEF . . .	12, 13
97 To construct a square equal in area to a given figure ABCDEFGH . . .	12, 13
98 To construct a parallelogram on a given base AB, equal in area to a given square CDEF . . .	14, 15
99 To construct a square equal in area to a given trapezium ABCD . . .	14, 15
100 To reduce or increase the number of sides of a regular figure, without changing the area . . .	14, 15
101 To construct a square, circle, or regular polygon, equal in area to two given similar figures . .	14, 15
102 To find the difference in area between two given squares . . .	14, 15
103 To construct a spiral curve, its greatest diameter being given, . .	16, 17

ART EDUCATION,

SCHOLASTIC AND INDUSTRIAL.

By WALTER SMITH,

Art Master, London; late Head Master of the Leeds School of Art and Science and Training School for Art Teachers; now Professor of Art Education in the City of Boston Normal School of Art, and State Director of Art Education, Massachusetts.

One Volume, Large Octavo. Bevelled Boards and Gilt Centre. With many Plain and Colored Illustrations. Price, in cloth, \$5.00.

THIS important work contains chapters on the first principles of Industrial Art; upon Design, Surface Decoration, Relief Ornament, Modelling, Casting and Carving, Architectural Enrichments, Symbolism in Art and Architecture; upon Art Education in the Common Schools, Schools of Art and Industrial Drawing; a description and comparison of French, English, and German methods of Industrial Art Study. It also includes chapters on designing, fitting, lighting, and furnishing with examples, Schools of Art and Industrial Drawing-Classes.

The book is illustrated with plans, sections, and elevations of Schools of Art and Art Institutions of America and England; original designs for schools; sketches of fitting, modes of lighting and seating class-rooms for Art Study, &c.; together with illustrations of choice specimens of antique house-furniture, glassware, and porcelain, colored plates of tiled floors and wood parquetry, a diagram of color, &c.

The volume has also several Appendixes, containing lists of examples for Art Study, their cost, and where they may be obtained, examples of examination-papers for various grades of Art Study, programmes of several European Schools of Art, Methods of Study, &c.

The foregoing incomplete statement of the contents of this book indicates its thorough and comprehensive character, and its admirable fitness to deepen and direct the popular interest already existing in Art and Art Study. The high reputation of the author as an Art Master, who has planned and furnished many Schools of Art, is a sufficient guaranty of the scientific accuracy and the practical usefulness of the work.

JAMES R. OSGOOD & CO., Publishers, Boston.

Drawing Copies

OF

Standard Reproductions and Original Designs,

FOR

PUBLIC SCHOOLS, DRAWING-CLASSES, AND SCHOOLS OF ART IN AMERICA.

EDITED AND DESIGNED BY
WALTER SMITH,
Art Master, London ; State Director of Art Education, Massachusetts.

THIS series of examples is issued in co-operation with the movement in favor of Art Instruction in America, and presents, for the purposes of study in classes and schools, an art-educational series equal to the best used in European Art Schools.

The examples comprised in it have been selected with reference to the needs of students who require practice in many subjects, and consist of an arrangement from the outline copies of British and other schools, together with new designs, in progressive order, and contain about six hundred examples.

The subjects illustrated are, —

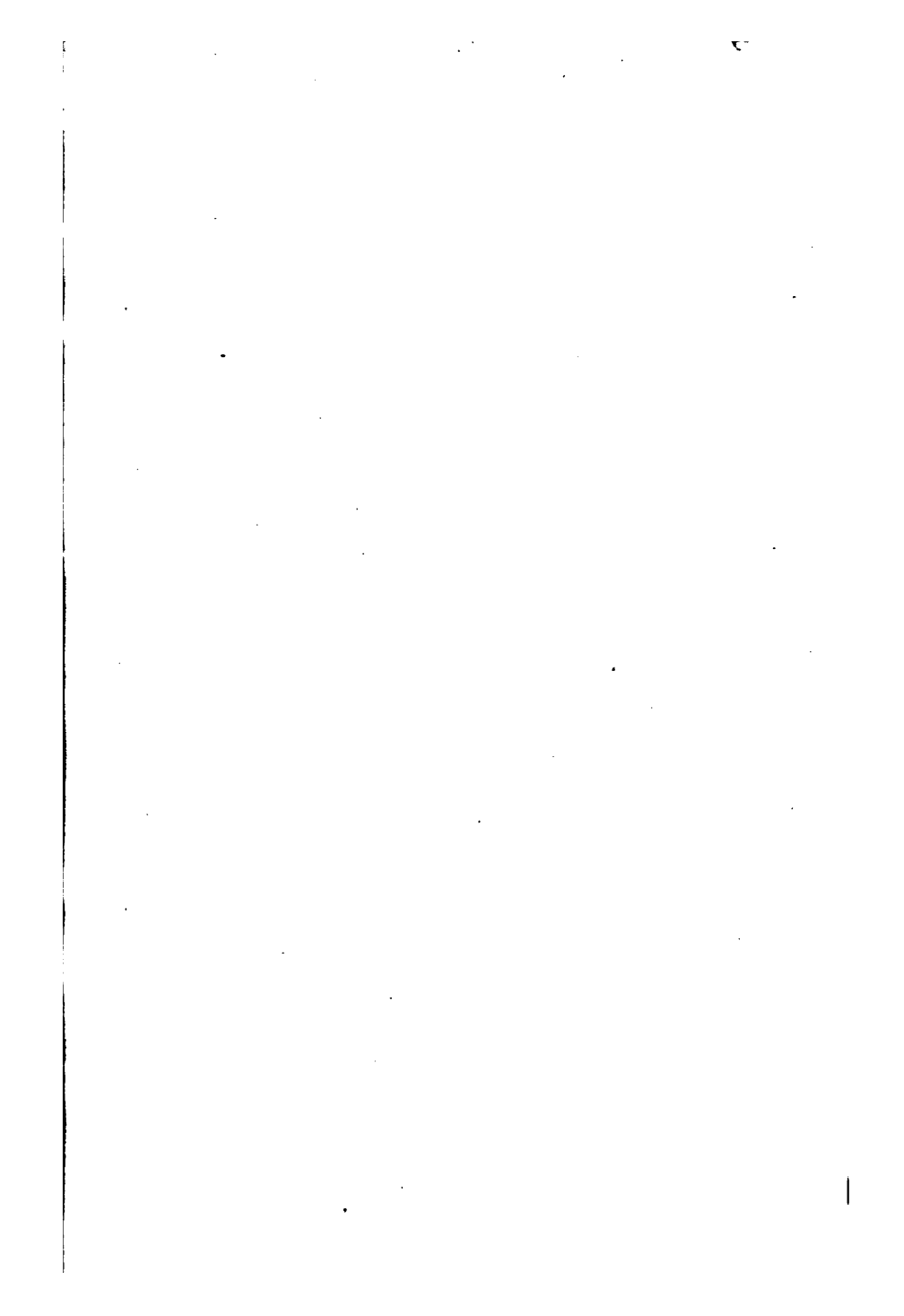
1. Ornamental Outlines, from Modern Design and the Antique.
2. Foliage and Flowers.
3. Greek and Roman Vases and Pottery Forms.
4. Model Drawings.
5. Conventionalized Foliage, from Nature, and Geometric Arrangements.
6. Animal Forms.
7. The Human Figure.

The series is elegantly printed on heavy plate paper, and is comprised in four parts, each part complete in itself, and issued in stiff paper wrapper.

The outline reproductions are of such works as have been tested and found valuable in European Schools, comprising, among others, Dyce's School of Design Drawing-Book, Hulme's Foliage, Albertolli's Outlines, Morghen's Human Figures, Delarue's Common Objects and Animal Forms, Wallis's Drawing-Book, and other standard works ; while the Original Designs have been carefully prepared by the Editor, Mr. WALTER SMITH, State Director of Art Education, Massachusetts.

In Four Parts, price \$5.00 each.

JAMES R. OSGOOD & CO., Publishers,
124 Tremont Street, Boston.





JUN 20 1884

SEP 2 1884

OCT 7 1893

Wm. York

DEC 9 1893





JUN 20 1884

SEP 20 1884

OCT 7 1893

Wm. York

DEC 9 1893